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MEMORY ABILITIES: A FACTOR ANALYSIS

By

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While I benefited greatly from the assistance acknowledged above, the responsibility for statements contained in this report is mine; the opinions and conclusions are my own and are not to be construed as necessarily reflecting the views or endorsement of the Educational Testing Service, the Navy Department, or the Air Force.

H. Paul Kelley

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Chapter I

DEFINITION OF THE PROBLEM

Introduction

One of the purposes of psychology is to attempt to understand and account for observable individual differences in behavior in terms of a limited number of concepts or ideal constructs. The names given to these constructs are unimportant; historically, some of them have been termed "abilities." Accepting this terminology, one can consider abilities as being, in a sense, determiners of behavior, in that the presence of an ability in an individual enables him to perform some task successfully, while the absence of that ability renders the performance of that task unsuccessful. It also may be considered that an individual's performance on a task or "test" is determined in part by the abilities that are called for by the test and in part by the degree to which the individual himself possesses these abilities. It is desirable that the definitions of these constructs or abilities be based on behaviors and relations between behaviors which can be observed experimentally.

With acceptance of the model established by these definitions, it becomes necessary to find some method for analysis of the observed test performance into the two components indicated above. Factor analysis is a refined technique for the study of individual differences. One of its aims is to obtain a measure of the extent to which each ability is called for by each test; the factor "loadings" computed from the analysis are such numerical measures. It is to be noted that in factor analysis, as in all other methods for the investigation of individual differences, two abilities are differentiated from each other only when (i) there is fairly wide variation of both abilities from subject to subject in the population being measured, (ii) the degree of correlation between these two abilities is fairly low in the population being tested, and (iii) measures or tests are being used in which successful performances depend to different degrees upon each of the two abilities.

If an ability model such as is presented above is to be used to describe behavior, the question arises as to the number and nature of abilities that are necessary to account for human behavior. For convenience, behavior may be roughly classified into certain groups, or domains, such as the memory domain or the reasoning domain. Since much, if not most, human behavior involves memory of one kind or another, it becomes important to know the

factorial structure of the memory domain. The simplest possible structure that could exist would be a general factor of memory; i.e., memory would be a unitary trait. Is this true? It seems very unlikely both on the basis of one's own personal experience and on the basis of past psychological experimentation. The following questions may then be raised. Is the memory function differentiated for the several modalities of presentation? Does immediate memory involve the same ability as that involved in the memory of past experience even when the subject makes no special effort to remember the experience? Is relatively immediate intentional retention a unitary trait; if not, how does it vary—e.g., with content or type of material, mode of presentation, modality of presentation, the psychological processes involved?

Results of Previous Research

What light has been shed on the problem of memory ability or abilities by previous research? Early studies, such as those by Hawkins (1897) and Worcester (1925), suggested that there are individual differences in the use of visual and auditory images for memory purposes, but that subjects who learn easily by visual presentation also tend to learn easily by auditory presentation. In general, transfer of training studies, such as that by Sleight (1911), seemed to provide no evidence to support the hypothesis of a general memory function; i.e., there seemed no *general* memory improvement as a result of practice. Additional results from noncorrelational studies which are relevant include those reported by Austin (1921), Bartlett (1932), Davis and Moore (1935), Ebbinghaus (1913), Hollingworth (1913), Katona (1940), Klüver (1933), and Kuhlmann (1908).

Many correlational studies have dealt, at least in part, with measures of memory ability; among the more important reports of results are these by Achilles (1920), Bennett (1916), Blankenship (1938), French (1953), Grant (1932), Guillet (1917), Henmon (1912), Husband (1939), Johannsen et. al. (1932), Jones and English (1926), Knott et. al. (1952), McGeoch (1928), U. S. War Department (1949–1950), Welborn and English (1937), and Wissler (1901).

Factor analysis studies have contributed the most direct evidence concerning memory abilities. Among the more important studies which utilized general-factor methods of analysis are those reported by Abelson (1911), Anastasi (1930, 1932), Bolton (1931), Carey (1914–1915, 1915–1917), Carothers (1921), Eysenck and Halstead (1945), Garrett (1928), Holzinger (1938), Holzinger and Harman (1938), Holzinger and Swineford (1939), T. L. Kelley (1928), Krueger and Spearman (1907), and Swineford and Holzinger (1942). The results of early studies by Spearman and his students were summarized by Spearman (1927), while later studies by this group were summarized by Spearman and Jones (1950). Vernon (1940, 1950) and Burt (1949) also commented upon the conclusions of general-factor studies.

Factor analysis studies which utilized multiple-factor analysis to obtain important information about the memory domain include those reported by Brener (1940), Bryan (1934), Carlson (1937), Fruchter (1953), Garrett (1938), Guilford (1947), Jones (1951), Karlin (1941, 1942), H. P. Kelley (1951), Rimoldi (1948), Taylor (1947), Thurstone (1938a, 1938b, 1949, 1951), Thurstone and Thurstone (1941), U. S. War Department (1949–1950), Woodrow (1939), Zachert and Friedman (1953), and Zimmerman (1953). The results of the early studies were summarized by Wolfle (1940), and Thurstone (1945, 1947b, 1951) commented a number of times on the conclusions he drew from factor studies from his laboratory.

French (1951), in his excellent summary and synthesis of factor studies, suggested four memory factors which he termed Associative or Rote Memory, Musical Memory, Span Memory, and Visual Memory. In the light of the studies reviewed, French raised the following questions.

(i) Is Span Memory distinct from Rote Memory?

(ii) Will Rote Memory decompose into two factors, recall and recognition?

(iii) Is Musical Memory distinct from Rote Memory? (The further question might be raised as to whether this factor is restricted to music or whether it might not instead be common to a larger class of auditory tasks.)

(iv) What is the nature of Visual Memory?

In general there seem to be three main points of weakness in previous factor studies in the domain of memory ability. The first weakness which might be pointed out is that many studies were too narrow to reveal the extent of and interdependencies among the different factors in the memory domain; the studies used too few memory tests; i.e., the sample of types of memory tasks was too small to delimit clearly the factors. This narrowness was largely due to the fact that most of the studies were primarily concerned with problems in domains other than that of memory.

A second point of weakness which has been fairly common concerns the matter of experimental dependence among tests in the battery. Thurstone (1947a) has shown that when tests in a battery are experimentally dependent upon one another, the factor structure is seriously disturbed, with one or more extra factors being added to the structure. Since such an extra factor usually involves only a very small number of tests, the rotational and interpretational problems are made much more complex, especially if there are several such factors in a single study.

The third weakness is the lack of reference tests in some of the analyses; when such tests are omitted from the analysis, the rotation of the factors to meaningful positions and their subsequent interpretation is rendered more difficult and questionable.

Questions To Be Answered

Of the many questions which have been raised by the studies mentioned above, this study will seek to find answers to the following.

(i) Is Rote Memory a distinct, separate factor?
(ii) Is Meaningful Memory a distinct, separate factor?
(iii) Is Span Memory a distinct, separate factor?
(iv) Are these three factors independent of the modality of presentation of the test material?
(v) Are these three factors independent of the type of test material used?
(vi) Is Visual Memory a distinct, separate factor?
(vii) Are these four factors independent of the method used in testing retention?

To obtain information that would be of help in answering these questions, the nature of four memory factors was postulated as follows.

1. *Rote Memory.* The ability to recall learned, meaningless material.
2. *Meaningful Memory.* The ability to recall learned, meaningful material.
3. *Span Memory.* The ability to recall perfectly a series of unrelated items after only one presentation of the series.
4. *Visual Memory.* The ability to recall material learned by the formation of an image of a whole visual field.

Additional factors might have been postulated, such as Auditory Memory (of one or more types) and Incidental Memory (of one or more types), but no attempt was made to investigate such factors in this study.

On the basis of the four hypotheses formulated above, a battery of memory tests was constructed by the author. Each test was intended to measure primarily one of these four factors, with an attempt being made to vary the task as much as possible within the limits of the stated hypotheses. Both visually presented and auditorily presented span tests were included in the battery. Although no Auditory Memory factor was expected in this battery, several auditorily presented nonspan tests were included in order that such a factor might be identified if it should appear. Both verbal and nonverbal tests were constructed in the attempt to measure Meaningful Memory. Answer types included primarily the traditional recall and recognition types, together with some true-false and multiple-choice items. The tests will be described in detail in the following chapter.

In connection with the test construction, the following sources were helpful to the author: Bronner et. al. (1932), Cook (1947), Guilford (1947), Guilford and Dallenbach (1925), Sharp (1949), U. S. Office of Strategic Services (1948), Wechsler (1945), and Whipple (1910).

Chapter II

DESCRIPTION OF THE TEST BATTERY

Introduction

The descriptions of the experimental tests below are divided into groups in terms of the factor for which each test was designed as a primary measure. The tests are numbered consecutively in the order in which they are listed below; this is not the order in which they were administered. The order and the details of the administration are discussed in Chap. III. All of the memory tests were administered as group tests.

Rote Memory Tests

1. Recognition Test I (Syllables)

Type of task: The examinee must indicate for each syllable in the test list whether or not that syllable was in the list of syllables which he studied previously.

Number of items: 18 syllables to be recognized from list of 36.

Time: 1-minute study time.

Score: Number correct.

2. Recognition Test II (Words)

Type of task: The examinee must indicate for each word in the test list whether or not that word was in the list of words which he heard previously. The words used are two-syllable nouns which are unrelated to each other.

Number of items: 25 words to be recognized from list of 50.

Time: Words read aloud at rate of approximately 2 seconds each.

Score: Number correct.

3. Recognition Test III (Figures)

Type of task: The examinee must indicate for each geometric figure or symbol in the test group whether or not that figure was in the group of figures which he studied previously.

Number of items: 40 figures to be recognized from group of 80.

Time: 1-minute study time.

Score: Number correct.

4. Memory for Syllables Test I

Type of task: The examinee must learn pairs of nonsense syllables so that when he is presented with the first syllable of a pair he can reproduce the second syllable.

Number of items: 2 parts, administered consecutively; 6 pairs per part.

Time: 1-minute study time for each part.

Score: Number correct.

5. Memory for Syllables Test II

Type of task: This test is a parallel form of test 4, Memory for Syllables Test I, using different nonsense syllables. The examinee must learn pairs of nonsense syllables so that when he is presented with the first syllable of a pair he can reproduce the second syllable.

Number of items: 2 parts, administered consecutively; 6 pairs per part.

Time: 1-minute study time for each part.

Score: Number correct.

6. Memory for Numbers Test

Type of task: The examinee must learn pairs of words and numbers so that when he is presented with the word of a pair he can reproduce the number.

Number of items: 2 parts, administered consecutively; 12 word-number pairs per part.

Time: 1-minute study time for each part.

Score: Number correct. (Owing to an error in the preparation of the test, the first part had only 10 possible correct answers, so the total possible score was 22 rather than 24.)

7. Memory for Words Test I (Unrelated words)

Type of task: The examinee must learn pairs of unrelated one-syllable nouns so that when he is presented with the first word of a pair he can reproduce the second word.

Number of items: 2 parts, administered consecutively; 10 pairs per part.

Time: Pairs read aloud twice at rate of approximately 2 seconds per pair, then first word of each pair read, allowing ample time for recording of responses.

Score: Number correct.

Meaningful Memory Tests

8. Memory for Words Test II (Related words)

Type of task: The examinee must learn pairs of related nouns so that

when he is presented with the first word of a pair he can reproduce the second word.

Number of items: 2 parts, administered consecutively; 25 pairs per part.

Time: 45-second study time for each part.

Score: Number correct.

9. Sentence Completion Test

Type of task: The examinee must learn a group of unrelated sentences so that when he is presented with a sentence with one word omitted he can reproduce the missing word. When studying the sentence, the examinee does not know which word will be omitted.

Number of items: 40 sentences.

Time: 4-minute study time, then delay of approximately 10 minutes (during which test 20, Memory for Instructions Test, is administered) before testing.

Score: Number correct.

10. Memory for Relations Test

Type of task: The examinee must learn a group of 3×3 progressive matrices of varied content (including letters, numbers, names of months, and geometric designs) so that when he is presented with the upper left-hand cell of a matrix he can reproduce whichever of the other cells is called for. (Actually, only cells 5, 6, 8, and 9 were required as responses; cells 2, 3, 4, and 7 were not tested.)

Number of items: 14 matrices.

Time: 6-minute study time.

Score: Number correct.

11. Consequences Test I (Nonverbal)

Type of task: The examinee is presented with several pairs of cartoon-type sketches, each pair being the first two panels of a three-panel sequence. After studying these pairs, the examinee is then presented with only the first picture of each sequence, and he must select from three choices the third picture which correctly completes that sequence.

Number of items: 18 sequences.

Time: 1-minute study time.

Score: Number correct.

12. Consequences Test II (Verbal)

Type of task: The examinee hears several pairs of sentences read aloud; the first sentence in each pair states a condition and the second sentence of the pair states a consequence of that condition. When the first sentence of each

pair is read again, the examinee must correctly reproduce in his own words the consequence to that condition.

Number of items: 20 pairs of sentences.

Time: Approximately 2.5-minute reading time for the 20 pairs.

Score: Number of ideas correct.

13. Memory for Limericks Test

Type of task: The examinee must learn a group of limericks so that when he is presented with the first four lines of a limerick he can correctly reproduce the idea and key words of the fifth line.

Number of items: 30 limericks.

Time: 5-minute study time.

Score: Number of key ideas correct.

14. Memory for Ideas Test

Type of task: After hearing a brief, one-paragraph story, "The Marble Statue," the examinee must reproduce it in his own words. See Bronner et al. (1932, p. 81).

Number of items: 67 idea units.

Time: Approximately 50-second reading time for the story.

Score: Number of idea units reproduced.

Span Memory Tests

15. Number Span Test I (Auditory)

Type of task: After hearing a sequence of digits, the examinee must reproduce the sequence. The sequences range in length from four to twelve digits.

Number of items: 2 sequences of each length, making 18 sequences in all.

Time: Approximately 1-second-per-digit reading time.

Score: Number of sequences completely correct.

16. Number Span Test II (Visual)

Type of task: After seeing a sequence of digits, the examinee must reproduce the sequence. The sequences range in length from four to twelve digits. Each sequence is presented one digit at a time by flash cards on a display stand.

Number of items: 2 sequences of each length, making 18 sequences in all.

Time: Approximately 1.5-2.0-second-per-digit display time.

Score: Number of sequences completely correct.

17. Letter Span Test I (Visual)

Type of task: After seeing a sequence of letters, the examinee must repro-

duce the sequence. The sequences range in length from three to eleven letters. Each sequence is presented one letter at a time by flash cards on a display stand.

Number of items: 2 sequences of each length, making 18 sequences in all.

Time: Approximately 1.5-2.0-second-per-letter display time.

Score: Number of sequences completely correct.

18. Letter Span Test II (Auditory)

Type of task: After hearing a sequence of letters, the examinee must reproduce the sequence. The sequences range in length from three to eleven letters.

Number of items: 2 sequences of each length, making 18 sequences in all.

Time: Approximately 1-second-per-letter reading time.

Score: Number of sequences completely correct.

19. Sentence Span Test

Type of task: After hearing a sentence, the examinee must reproduce it. The sentences range in length from 18 to 33 syllables.

Number of items: 16 sentences.

Time: 3.0-6.5-second reading time per sentence, depending on length of sentence.

Score: Number of sentences completely correct.

20. Memory for Instructions Test

Type of task: After hearing a set of instructions (e.g., "Check the *9*. Cross out the *R*."), the examinee must carry out the instructions. The sets range in length from two to five instructions.

Number of items: 16 sets of instructions.

Time: Approximately 1.5-second per instruction reading time.

Score: Number of sets of instructions performed completely correctly.

Visual Memory Tests

21. Reproduction of Visual Designs Test

Type of task: After seeing a geometric design on a flash card, the examinee must reproduce that design.

Number of items: 10 designs.

Time: 5-second display time for each design.

Score: 2 points for each correct design, 1 point for each design with only minor deviations from the correct design.

22. Map Memory Test I (Reproduction)

Type of task: After studying a product-type map of a fictional country, the examinee must reproduce the map.

Number of items: 31 features.

Time: 2-minute study time.

Score: In general, 1 point for presence of each feature, and 1 point for the correct location of that feature; total possible score was 56. Full credit was given if a name was reproduced instead of a symbol.

23. Map Memory Test II (Verbal recall)

Type of task: After studying a map of a section of town and countryside, the examinee must answer multiple-choice questions about the area portrayed by the map.

Number of items: 14 questions.

Time: 30-second study time.

Score: Number correct.

24. Map Memory Test III (Recognition)

Type of task: The examinee must learn the map of an area of countryside so that when he is presented with five representations of a section of that map he can indicate which is the correct representation.

Number of items: 12 five-choice items.

Time: 3-minute study time.

Score: Number correct.

Special Tests

25, 26, 27. Meaningful Memory: Picture, Paragraph, Number

Type of task: This test was divided into three sections, each section being scored separately. Hence, the sections were treated as three separate tests. In the Picture section, the examinee must learn the details of a sketch representing a Venetian scene so that when he is presented with a sketch of a similar Venetian scene he can answer true-false questions about the similarities and differences of the two pictures. In the Paragraph section, the examinee must learn a long encyclopedia-type article about a country so that he can answer true-false questions about it. In the Number section, the examinee must learn detailed information about inventories in two stores so that he can answer multiple-choice questions about them.

Number of items: Picture section, 30 true-false items; Paragraph section, 30 true-false items; Number section, 15 five-choice items.

Time: 15-minute study time, 5 minutes for each section; then delay of approximately 20-24 minutes during which tests 18 (Letter Span Test II)

TABLE 1

Classification of the 27 Memory Tests by Method of Testing Retention

Recognition Recall	Completion Recall		Free Recall
	Paired Associates	Other	
1. Recognition I (Syllables)	4. Memory for Syllables I	9. Sentence Completion	14. Memory for Ideas
2. Recognition II (Words)	5. Memory for Syllables II	10. Memory for Relations	15. Number Span I (Auditory)
3. Recognition III (Figures)	6. Memory for Numbers	11. Consequences I (Nonverbal)	16. Number Span II (Visual)
24. Map Memory III (Recognition)	7. Memory for Words I (Unrelated)	12. Consequences II (Verbal)	17. Letter Span I (Visual)
	8. Memory for Words II (Related)	13. Memory for Limericks	18. Letter Span II (Auditory)
		23. Map Memory II (Verbal)	19. Sentence Span
		25. Mean. Memory: Picture	20. Memory for Instructions
		26. Mean. Memory: Paragraph	21. Reproduction of Visual Designs
		27. Mean. Memory: Number	22. Map Memory I (Reproduction)

and 21 (Reproduction of Visual Designs) are administered; then 15-minute testing time, 5 minutes for each section.

Score: Number correct.

These three tests, developed by the Educational Testing Service, were not considered to be primary measures of one or more of the four hypothesized factors. However, in the light of the hypotheses it was expected that the Picture section would have its highest loading on Visual Memory, and that the Paragraph and Number sections would be highest on Meaningful Memory.

Table I gives a classification of the memory tests by method of testing. The Recognition Recall tests merely require the examinee to recognize stimuli which he has previously experienced; the Completion Recall tests require the examinee to reproduce a part of each previously examined stimulus when he is presented with the remainder of that stimulus; the Free Recall tests require the examinee to reproduce completely the previously experienced stimuli.

TABLE 2

Classification of the 27 Memory Tests by
Type of Material and Modality of Presentation

Verbal Material		Nonverbal Material	
Visual Modality	Auditory Modality	Visual Modality	Auditory Modality
1. Recognition I (Syllables)	2. Recognition II (Words)	3. Recognition III (Figures)	(None)
4. Memory for Syllables I	7. Memory for Words I (Unrelated)	10. Memory for Relations	
5. Memory for Syllables II	12. Consequences II (Verbal)	11. Consequences I (Nonverbal)	
6. Memory for Numbers	14. Memory for Ideas	21. Reproduction of Visual Designs	
8. Memory for Words II (Related)	15. Number Span I (Auditory)	25. Meaningful Memory: Picture	
9. Sentence Completion	18. Letter Span II (Auditory)		
13. Memory for Limericks	19. Sentence Span		
16. Number Span II (Visual)	20. Memory for Instructions		
17. Letter Span I (Visual)			
26. Meaningful Memory: Paragraph			
27. Meaningful Memory: Number			
*22. Map Memory I (Reproduction)		*22. Map Memory I (Reproduction)	
23. Map Memory II (Verbal)		23. Map Memory II (Verbal)	
24. Map Memory III (Recognition)		24. Map Memory III (Recognition)	

*Although these Map Memory tests were primarily nonverbal, the maps did con tain some verbal material, especially Map Memory I.

Table 2 gives a classification of the memory tests by type of material and modality of presentation. It will be remembered that this study did not attempt to investigate auditory memory; however, some auditory tests were included in the study in order to estimate the generality of the hypothesized factors of Rote, Meaningful, and Span Memory.

Reference Tests

It seems quite evident that the tests described above measure more than just memory abilities. In order to find out what additional abilities are being measured, so that these sources of variance can be recognized and taken into account in the interpretation of the memory factors, thirteen additional tests were added to the test battery. Henceforth, these tests will be called reference tests. All of these tests were chosen from the U. S. Air Force Airman Classification Battery; detailed descriptions may be found in the U. S. Army Air Forces Aviation Psychology Program Research Reports by Cook (1947), Guilford (1947), and Melton (1947). Brief descriptions of these tests will be given below. Scores on these tests are reported by the Air Force in terms of stanines. Since all of the examinees had previously taken these tests, it was unnecessary to readminister them with the memory tests.

28. Instrument Comprehension (CI 616 C-C2)

Type of task: The examinee is presented with drawings of two instruments, a compass and an artificial horizon, followed by five photographs showing an airplane in different positions; he must choose the picture which is in agreement with the two instrument readings.

Number of items: 60.

Time: 15 minutes. This test is fairly highly speeded.

Score: $R - (W/4)$.

Factor content: Spatial Relations, Visualization, Reasoning II.

29. Mechanical Principles (CI 903 B)

Type of task: The examinee must answer questions concerning mechanical principles and devices which are illustrated by means of pictures and diagrams.

Number of items: 40.

Time: 20 minutes. This test is not stated to be speeded.

Score: $R - (W/2) + 20$.

Factor content: Deduction, Mechanical Experience, Visualization, Spatial Relations.

30. Rudder Control (CM 120 C)

Type of task: This test requires the manipulation of rudder pedals to bring a cockpit into a condition of equilibrium. The examinee is placed in a

model cockpit and instructed to keep a sighting bar on the fuselage in front of him pointed at a target.

Number of trials: 6.

Time: 1 minute per trial.

Score: Total time cockpit is pointed directly at target.

Factor content: Psychomotor Coordination, Visualization, Pilot or Flying Interest.

31. Complex Coordination (CM 701 E)

Type of task: The examinee must respond to complex perceptual signals by making coordinated movements of airplane controls. A pattern of three red lights is presented to the examinee, who must then manipulate the stick and rudder controls so as to light corresponding green lights; as soon as all lights are matched, a new stimulus pattern is presented.

Number of items: Varies with examinee.

Time: 8 minutes.

Score: Number of patterns correctly matched.

Factor content: Psychomotor Coordination, Spatial Relations.

32. Arithmetic Reasoning (CI 206 C)

Type of task: The examinee must solve mathematical problems which are stated verbally; many of the problems are couched in aviation terms.

Number of items: 30.

Time: 35 minutes. This test is not stated to be speeded.

Score: $2R - (W/2)$.

Factor content: Numerical Facility, Deduction, Verbal Comprehension.

33. Reading Comprehension (CI 614 H)

Type of task: The examinee must make valid inferences from reading material as well as answer more direct questions about content. The paragraphs were taken from technical material, including texts on navigation, physics, map reading, astronomy, and airplane instruments.

Number of items: 8 paragraphs, 36 questions.

Time: 30 minutes. This test is fairly highly speeded.

Score: $2R - (W/2)$.

Factor content: Verbal Comprehension, Deduction, Numerical Facility.

34. Vocabulary (CI 604 B)

Type of task: The examinee must select synonyms for given words.

Number of items: 150.

Time: 15 minutes. This test is highly speeded.

Score: $R - (W/4)$.

Factor content: Verbal Comprehension.

35. Dial and Table Reading (CP 622–21 A)

Type of task: In the dial-reading section, the examinee is presented with seven dials such as might be found on the control panel of an airplane; he must answer questions concerning the readings on the dials. In the table-reading section he must answer questions which require the consultation of tables of figures.

Number of items: Dial-reading section, 57 questions on 10 sets of dials; table-reading section, 86 questions.

Time: Dial-reading section, 9 minutes; table-reading section, 15 minutes. This test is fairly highly speeded.

Score: $\frac{1}{2}(R - W)$.

Factor content: Perceptual Speed, Numerical Facility, Spatial Relations.

36. Spatial Orientation I (CP 501 B)

Type of task: At the top of each test page there is a large aerial photograph, with six circular photographs below it which are sections of it. The examinee must find the area in the large photograph that is the same as each of the small photographs.

Number of items: 49 items based on 9 large aerial photographs.

Time: 5 minutes. This test is highly speeded.

Score: $R - W + 20$.

Factor content: Perceptual Speed.

37. Coordinate Reading (CP 224 B)

Type of task: The examinee is presented with a circular graph which simulates an oscilloscope screen; this graph is graduated in degrees from 0° to 360°, and in concentric circles representing ten-mile intervals. Located within the circle are dashes representing target returns on the oscilloscope screen. The examinee must determine the bearing and range of each dash line from the center of the circle; the items are multiple choice in form.

Number of items: 85.

Time: 20 minutes. This test is speeded.

Score: Total number correct.

Factor content: Perceptual Speed, Numerical Facility, Spatial Relations.

38. Discrimination Reaction Time (CP 611 D)

Type of task: The examinee is presented with a visual stimulus pattern, consisting of one red and one green light, in which the principal element is the spatial relation of the two lights. He must make a differential response to this spatial arrangement by tripping one of four switches, the correct switch depending on the position of the red light with respect to the green one.

Number of items: 80 reactions, in 4 groups of 20 each.

Time: Varies with examinee.

Score: Total accumulated time between stimulus and correct response.
Factor content: Spatial Relations, Perceptual Speed, Finger Dexterity.

39. Spatial Orientation II (CP 503 B)

Type of task: Each test page contains a standard aviation map which is sectioned off into twelve squares; below the map are four aerial photographs of portions of the area portrayed in the map. The examinee must match the photographs to the proper sections of the map.
Number of items: 50 items based on 13 aerial maps.
Time: 18 minutes. This test is highly speeded.
Score: $R - W + 20$.
Factor content: Perceptual Speed, Visualization.

40. Numerical Operations (CI 702 B)

Type of task: The examinee must solve simple problems in addition, subtraction, multiplication, and division.
Number of items: Part I (addition and multiplication), 100 items; Part II (subtraction and division), 80 items.
Time: 5 minutes for each part. This test is highly speeded.
Score: $\frac{1}{2}(R - 3W)$.
Factor content: Numerical Facility.

These, then, were the 40 tests making up the test battery. The methods used in the collection and analysis of data will be discussed in the next chapter.

Chapter III

DATA COLLECTION AND ANALYSIS

Population

The sample used in this study was drawn from a population of U. S. Air Force pilot cadets who were entering basic pilot training. Six groups of cadets took the 27 memory tests; the total number of cadets tested was 480.

All of these men had previously taken the tests comprising the Air Force Classification Battery, so it was unnecessary to readminister the reference tests. However, reference-test scores were unavailable for some of the men; complete information was obtainable for 442, so only these subjects were used in this study.

The ages of the examinees ranged from 19 to 27 years, with a mean age of 21.6 years. All men had at least a high-school education; 103 (23.3%) had not been to college, 254 (57.5%) had been in college from one to three years, and 85 (19.2%) had taken four or more years of college work.

Testing Procedure

The testing was conducted by an Air Force testing team; the author was present at all testing sessions as an observer and adviser. Since the tests were administered by Air Force personnel, and since many Air Force tests were administered during the same sessions as the memory tests, there is reason to believe that the men considered the memory tests to be a regular part of the Air Force testing. It was hoped that the regular orientation statement made by the testing officer at the outset of the testing would provide sufficient motivation for the subjects to perform as well as possible.

The memory tests, bound together in three test booklets, were administered in the order shown in Table 3. The first six tests were in Test Book I; Test Book II contained the next nine tests; and the last twelve tests were in Test Book III.

Testing of each examinee was completed in a single day. In the morning testing session, three Air Force tests were administered; following this, the men were given a ten-minute rest period. Then they were given another Air Force test, followed by Book I of the memory battery. After Book I, there was a 2-3 minute rest period during the collection of Book I and the distribution of Book II. After the administration of Book II, the men were dismissed for lunch. In the afternoon testing session, the first test administered

TABLE 3

Order of Administration of Experimental Memory Tests

Order of Administration	Test Number	Identification Symbol	Test Name
1	13	Lim	Memory for Limericks Test
2	17	LSp-Vi	Letter Span Test I (Visual)
3	4	PSyl-1	Memory for Syllables Test I
4	11	Con-Nv	Consequences Test I
5	15	NSp-Au	Number Span Test I (Auditory)
6	7	PWd-U	Memory for Words Test I (Unrelated words)
7	1	RSyl	Recognition Test I (Syllables)
8	14	Ideas	Memory for Ideas Test (Story)
9	22	Map-Rp	Map Memory Test I (Reproduction)
10	16	NSp-Vi	Number Span Test II (Visual)
11	25	Pict	Meaningful Memory: Picture Test
12	26	Para	Meaningful Memory: Paragraph Test
13	27	Num	Meaningful Memory: Number Test
14	18	LSp-Au	Letter Span Test II (Auditory)
15	21	Dsg-Rp	Reproduction of Visual Designs Test
16	12	Con-V	Consequences Test II (Verbal)
17	2	RWd	Recognition Test II (Words)
18	6	PNum	Memory for Numbers Test
19	23	Map-Vb	Map Memory Test II (Verbal)
20	24	Map-Rc	Map Memory Test III (Recognition)
21	19	SenSp	Sentence Span Test
22	5	PSyl-2	Memory for Syllables Test II
23	3	RFig	Recognition Test III (Figures)
24	10	Rel	Memory for Relations Test
25	9	SComp	Sentence Completion Test
26	20	InsSp	Memory for Instructions Test
27	8	PWd-R	Memory for Words Test II (Related words)

TABLE 4

Descriptive Statistics of 40 Tests for 442 Subjects

Test	Maximum Possible Score	Mean	Standard Deviation	Final Communality Estimate
1. Recognition I (Syllables)	36	25.94	4.36	.32
2. Recognition II (Words)	50	42.88	4.37	.46
3. Recognition III (Figures)	80	62.61	6.37	.32
4. Memory for Syllables I	12	3.96	2.21	.63
5. Memory for Syllables II	12	5.31	2.87	.55
6. Memory for Numbers	22	10.60	4.75	.51
7. Memory for Words I (Unrelated words)	20	12.94	4.56	.59
8. Memory for Words II (Related words)	50	34.71	7.12	.54
9. Sentence Completion	40	19.64	6.64	.58
10. Memory for Relations	14	9.55	2.50	.39
11. Consequences I (Nonverbal)	18	15.21	2.50	.23
12. Consequences II (Verbal)	20	17.29	2.69	.34
13. Memory for Limericks	30	11.56	5.49	.71
14. Memory for Ideas	67	37.98	6.06	.55
15. Number Span I (Auditory)	18	7.40	2.23	.56
16. Number Span II (Visual)	18	8.69	2.45	.55
17. Letter Span I (Visual)	18	8.46	2.11	.59
18. Letter Span II (Auditory)	18	7.84	2.12	.63
19. Sentence Span	16	8.49	2.70	.47
20. Memory for Instructions	16	9.01	2.47	.29
21. Reproduction of Visual Designs	20	17.21	2.08	.52
22. Map Memory I (Reproduction)	56	42.89	5.72	.34
23. Map Memory II (Verbal recall)	14	5.28	1.95	.18
24. Map Memory III (Recognition)	12	7.31	2.28	.39
25. Meaningful Memory: Picture	30	23.18	3.27	.24
26. Meaningful Memory: Paragraph	30	23.17	2.93	.45
27. Meaningful Memory: Number	15	11.89	2.91	.35
28. Instrument Comprehension	9	5.36	2.02	.37
29. Mechanical Principles	9	5.38	1.98	.55
30. Rudder Control	9	5.63	1.79	.36
31. Complex Coordination	9	5.36	1.82	.44
32. Arithmetic Reasoning	9	5.63	2.00	.72
33. Reading Comprehension	9	5.52	1.86	.68
34. Vocabulary	9	4.99	1.97	.74
35. Dial and Table Reading	9	5.55	2.01	.64
36. Spatial Orientation I	9	4.93	1.94	.60
37. Coordinate Reading	9	4.75	2.14	.51
38. Discrimination Reaction Time	9	5.51	1.92	.31
39. Spatial Orientation II	9	5.41	2.03	.51
40. Numerical Operations	9	5.17	1.72	.66

was another Air Force test, then Book III of the memory battery. The men were allowed a ten-minute rest period between tests 5 and 3. The last test for the day was an Air Force test. All of the Air Force tests administered were unrelated to this study.

In addition to these tests, each man spent another full day taking only Air Force tests; three groups took those tests before taking the memory battery, while the other three groups took those tests after taking the memory tests.

Scoring the Tests

All 27 of the memory tests were independently scored twice; any discrepancy in score was eliminated by a third scoring of that test. Two of the tests—21 (Reproduction of Visual Designs) and 14 (Memory for Ideas)—were scored by the author; the other tests were scored by members of the scoring staff at the Educational Testing Service. The actual scoring methods used were reported in Chap. II. The single-digit stanine scores for the reference tests were furnished by the Air Force.

Table 4 gives the total possible score, the mean, and the standard deviation for each test in the battery.

No direct measurement of the reliability of the 27 memory tests was made; however, two types of indirect estimates were available. It can be shown that the reliability of a test is equal to or greater than the square of the correlation between that test and any other variable; also, the final communality estimate, h_j^2 , for each test j furnishes another lower-bound on the reliability coefficient, since $h_j^2 \leq r_{jj}$ (Thurstone, 1947a, p. 84). Since for every test in this battery the final communality estimate was greater than the square of the highest correlation coefficient for that test, the final communality estimates are included in Table 4 as lower-bounds on the reliability coefficients. For the purposes of a factor analysis study it is not necessary that a test be as reliable as it should be if it were to be used for the selection of individual examinees; with this in mind it can be seen that most of the tests in the battery are reliable enough to yield meaningful factorial results.

Reliability data on the 13 reference tests have been reported by Cook (1947), Guilford (1947), and Melton (1947).

Computation of Correlation Coefficients

The coefficients of correlation were computed on an IBM Card-Programmed Calculator, the only electronic computer available to the author at the time these computations were made. Table 5 presents the intercorrelations of the 40 tests.

Factor Analysis

The multiple-group method of factoring (Thurstone, 1947a, pp. 170-175)

was used to obtain the original unrotated orthogonal factor matrix. Expressed in matrix algebra, the method given by Thurstone is as follows.

R_{jk} = reduced correlation matrix (communality estimates in diagonal cells).

W = weight matrix defining groups.

(1) $R_{jk}W = Z$.

(2) $Z'W = W'R_{jk}W = T$.

(3) $Y \equiv$ a diagonal matrix with entries equal to $1/\sqrt{t_{ii}}$.

(4) $YTY = R_{pq}$ (i.e., Y is so defined that R_{pq} has unit diagonals).

(5) $R_{pq} = E'E$. R_{pq} is factored by the diagonal or triangularization method. (Thurstone, 1947a, pp. 101–105.)
E^{-1} is computed.

(6) $ZY = U$.

(7) $UE^{-1} = F$, where $R_{jk} = FF'$.

Dr. Ledyard R. Tucker pointed out that the steps involving the Y matrix could be eliminated, so the method reduced to the following steps.

(8) $R_{jk}W = Z$.

(9) $Z'W = W'R_{jk}W = T$.

(10) $T = B'B$. (T is factored by the diagonal method.)
B^{-1} is computed.

(11) $ZB^{-1} = F$, where $R_{jk} = FF'$.

In the equations given above, W defines the groups; the latter were chosen to represent clusters in the correlation matrix. A first examination of the matrix of intercorrelations resulted in the selection of seven groups; the weight matrix W_a defining these groups is shown in Table 6 as the first seven columns of W. The highest correlation coefficient in each column of the correlation matrix was chosen as the first communality estimate for that variable; these seven groups were factored out of the correlation matrix R_0 by the method outlined above. The result was a factor matrix, F_a.

A matrix of residual correlation coefficients R_a was then computed by the formula

(12) $$R_a = R_0 - F_aF_a' .$$

TABLE 5

Correlation Coefficients of 40 Tests for 442 Subjects: Matrix R_0

(Decimal points omitted)

Test	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1. RSyl	—	16	29	21	27	24	23	26	24	14	07	16	23	09	08	11	12	18	12	−02
2. RWd	16	—	18	15	27	38	42	36	30	04	05	23	10	18	11	19	09	15	08	−07
3. RFig	29	18	—	13	23	28	21	30	28	22	19	17	22	18	09	05	11	15	15	05
4. PSyl-1	21	15	13	—	51	42	39	31	32	15	08	20	25	14	21	28	23	28	23	08
5. PSyl-2	27	27	23	51	—	48	31	39	31	25	14	27	26	15	23	33	31	32	20	22
6. PNum	24	38	28	42	48	—	46	46	40	14	14	20	21	13	17	20	21	26	14	08
7. PWd-U	23	42	21	39	31	46	—	39	38	19	18	20	22	25	20	19	17	28	15	−05
8. PWd-R	26	36	30	31	39	46	39	—	52	14	16	34	26	20	10	23	19	18	09	04
9. SComp	24	30	28	32	31	40	38	52	—	17	18	41	48	34	15	16	13	19	07	11
10. Rel	14	04	22	15	25	14	19	14	17	—	22	13	17	12	19	20	13	15	11	09
11. Con-Nv	07	05	19	08	14	14	18	16	18	22	—	17	29	26	08	16	17	07	19	07
12. Con-V	16	23	17	20	27	20	20	34	41	13	17	—	40	31	11	13	15	16	24	07
13. Lim	23	10	22	25	26	21	22	26	48	17	29	40	—	53	16	23	23	29	36	10
14. Ideas	09	18	18	14	15	13	25	20	34	12	26	31	53	—	16	17	14	17	32	−03
15. NSp-Au	08	11	09	21	23	17	20	10	15	19	08	11	16	16	—	50	49	58	31	23
16. NSp-Vi	11	19	05	28	33	20	19	23	16	20	16	13	23	17	50	—	52	53	28	29
17. LSp-Vi	12	09	11	23	31	21	17	19	13	13	17	15	23	14	49	52	—	55	35	28
18. LSp-Au	18	15	15	28	32	26	28	18	19	15	07	16	29	17	58	53	55	—	39	22
19. SenSp	12	08	15	23	20	14	15	09	07	11	19	24	36	32	31	28	35	39	—	14
20. InsSp	−02	−07	05	08	22	08	−05	04	11	09	07	07	10	−03	23	29	28	22	14	—
21. Dsg-Rp	12	09	28	14	26	16	07	13	16	36	16	10	12	08	17	16	16	18	14	17
22. Map-Rp	14	21	27	19	29	30	20	27	23	26	23	11	11	17	08	15	15	12	−04	07
23. Map-Vb	17	10	15	13	20	21	08	22	21	17	10	16	11	11	13	20	11	11	03	05
24. Map-Rc	22	10	26	16	28	23	20	22	22	32	15	07	16	15	15	16	23	19	04	05
25. Pict	18	11	25	18	25	16	17	25	25	23	18	18	22	21	20	20	24	18	16	13
26. Para	26	17	18	17	22	20	24	26	25	19	17	23	32	22	09	15	18	19	22	01
27. Num	18	22	11	22	26	26	21	25	17	31	13	01	01	05	16	20	11	17	09	03
28. IComp	13	00	14	11	14	07	09	11	13	25	10	06	19	05	14	10	15	17	13	12
29. MPrin	01	01	16	−02	−00	03	07	05	05	15	01	09	15	15	13	05	06	13	17	−01
30. RCon	−04	−00	−05	−05	−09	−11	03	−03	−05	−04	−07	−03	−10	−07	10	03	12	10	−02	−02
31. CCord	09	02	05	02	05	10	04	08	04	21	08	05	10	10	20	13	18	20	−02	00
32. ArReas	17	14	16	13	17	14	11	17	22	36	18	22	33	17	20	25	16	23	27	08
33. RdComp	26	02	22	14	21	08	15	15	25	30	20	24	46	26	20	17	21	27	36	10
34. Vocab	26	03	20	26	29	15	14	18	25	18	25	26	59	34	22	21	30	31	45	13
35. DTRd	10	14	08	15	18	09	−00	09	15	37	17	12	13	07	16	17	15	20	10	07
36. SpaO-1	−01	05	07	09	18	08	−04	11	10	19	19	05	05	08	08	15	14	13	10	13
37. CRd	12	09	12	04	13	12	−01	08	14	29	14	06	10	06	14	14	09	11	09	03
38. DRTime	08	17	12	12	15	16	12	15	17	24	17	12	12	07	21	13	21	16	07	03
39. SpaO-2	10	00	16	04	15	09	−02	12	09	27	17	03	08	05	06	11	14	10	10	05
40. NumOp	12	12	04	15	20	15	04	19	20	27	18	15	25	10	16	24	16	14	14	11

TABLE 5 (cont.)
Correlation Coefficients of 40 Tests for 442 Subject: Matrix R_0
(Decimal points omitted)

Test	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1. RSyl	12	14	17	22	18	26	18	13	01	−04	09	17	26	26	10	−01	12	08	10	12
2. RWd	09	21	10	10	11	17	22	00	01	−00	02	14	02	03	14	05	09	17	00	12
3. RFig	28	27	15	26	25	18	11	14	16	−05	05	16	22	20	08	07	12	12	16	04
4. PSyl-1	14	19	13	16	18	17	22	11	−02	−05	02	13	14	26	15	09	04	12	04	15
5. PSyl-2	26	29	20	28	25	22	26	14	−00	−09	05	17	21	29	18	18	13	15	15	20
6. PNum	16	30	21	23	16	20	26	07	03	−11	10	14	08	15	09	08	12	16	09	15
7. PWd-U	07	20	08	20	17	24	21	09	07	03	04	11	15	14	−00	−04	−01	12	−02	04
8. PWd-R	13	27	22	22	25	26	25	11	05	−03	08	17	15	18	09	11	08	15	12	19
9. SComp	16	23	21	22	25	25	17	13	05	−05	04	22	25	25	15	10	14	17	09	20
10. Rel	36	26	17	32	23	19	31	25	15	−04	21	36	30	18	37	19	29	24	27	27
11. Con-Nv	16	23	10	15	18	17	13	10	01	−07	08	18	20	25	17	19	14	17	17	18
12. Con-V	10	11	16	07	18	23	01	06	09	−03	05	22	24	26	12	05	06	12	03	15
13. Lim	12	11	11	16	22	32	01	19	15	−10	10	33	46	59	13	05	10	12	08	25
14. Ideas	08	17	11	15	21	22	05	05	15	−07	10	17	26	34	07	08	06	07	05	10
15. NSp-Au	17	08	13	15	20	09	16	14	13	10	20	20	20	22	16	08	14	21	06	16
16. NSp-Vi	16	15	20	16	20	15	20	10	05	03	13	25	17	21	17	15	14	13	11	24
17. LSp-Vi	16	15	11	23	24	18	11	15	06	12	18	16	21	30	15	14	09	21	14	16
18. LSp-Au	18	12	11	19	18	19	17	17	13	10	20	23	27	31	20	13	11	16	10	14
19. SenSp	14	−04	03	04	16	22	09	13	17	−02	−02	27	36	45	10	10	09	07	10	14
20. InsSp	17	07	05	05	13	01	03	12	−01	−02	00	08	10	13	07	13	03	03	05	11
21. Dsg-Rp	—	30	15	28	22	07	27	28	25	−06	16	21	23	15	18	19	25	17	32	11
22. Map-Rp	30	—	20	31	21	20	25	17	11	−06	13	19	14	17	26	28	22	20	30	19
23. Map-Vb	15	20	—	21	05	07	17	07	01	04	10	10	08	05	10	12	08	04	13	03
24. Map-Rc	28	31	21	—	27	19	25	23	17	−06	14	18	24	15	21	20	26	13	32	10
25. Pict	22	21	05	27	—	19	22	20	10	00	18	16	21	24	22	15	16	23	16	22
26. Para	07	20	07	19	19	—	23	06	11	−10	01	27	37	42	18	10	16	16	19	23
27. Num	27	25	17	25	22	23	—	07	−01	−05	07	19	14	07	17	15	13	19	12	24
28. IComp	28	17	07	23	20	06	07	—	30	14	29	29	32	20	35	26	37	27	35	15
29. MPrin	25	11	01	17	10	11	−01	30	—	25	32	45	40	17	18	14	29	19	32	04
30. RCon	−06	−06	04	−06	00	−10	−05	14	25	—	30	03	05	−03	05	02	−01	05	01	−09
31. CCoord	16	13	10	14	18	01	07	29	32	30	—	19	18	10	27	24	23	34	29	15
32. ArReas	21	19	10	18	16	27	19	29	45	03	19	—	54	35	38	05	36	23	31	48
33. RdComp	23	14	08	24	21	37	14	32	40	05	18	54	—	61	28	15	23	21	31	23
34. Vocab	15	17	05	15	24	42	07	20	17	−03	10	35	61	—	21	19	16	21	21	27
35. DTRd	18	26	10	21	22	18	17	35	18	05	27	38	28	21	—	44	57	36	33	52
36. SpaO-1	19	28	12	20	15	10	15	26	14	02	24	05	15	19	44	—	39	32	42	21
37. CRd	25	22	08	26	16	16	13	37	29	−01	23	36	23	16	57	39	—	28	38	39
38. DRTime	17	20	04	13	23	16	19	27	19	05	34	23	21	21	36	32	28	—	23	29
39. SpaO-2	32	30	13	32	16	19	12	35	32	01	29	31	31	21	33	42	38	23	—	15
40. NumOp	11	19	03	10	22	23	24	15	04	−09	15	48	23	27	52	21	39	29	15	—

TABLE 6

Weight Matrix W

(All empty cells indicate zero weights.)

Test	Factor										
	A_1	B_1	C_1	D_1	E_1	F_1	G_1	H_1	I_1	J_1	K_1
1. RSyl								−1			
2. RWd									1	1	
3. RFig									1		
4. PSyl-1							1		−1		
5. PSyl-2							1			−1	
6. PNum							1				
7. PWd-U							1			1	
8. PWd-R							1		1		
9. SComp							1	1	1		
10. Rel				1							1
11. Con-Nv								1			
12. Con-V								1	1		
13. Lim	1							1			
14. Ideas								1	1	1	
15. NSp-Au						1					
16. NSp-Vi						1					
17. LSp-Vi						1				−1	
18. LSp-Au						1					
19. SenSp						1				1	−1
20. InsSp										−1	
21. Dsg-Rp								1			
22. Map-Rp											1
23. Map-Vb											
24. Map-Rc											1
25. Pict											1
26. Para	1							−1			
27. Num											1
28. IComp			1	1	1						
29. MPrin				1	1						
30. RCon					1						
31. CCoord					1						1
32. ArReas		1		1							
33. RdComp	1			1							
34. Vocab	1								−1		
35. DTRd		1	1								
36. SpaO-1			1								
37. CRd			1	1							
38. DRTime											1
39. SpaO-2			1	1							
40. NumOp		1									

An examination of R_a revealed four groups or clusters; W_b , which defines these groups, is shown in Table 6 as the last four columns of W. The communalities were re-estimated in the same manner as before; then these four groups were factored out of R_a , giving factor matrix F_b .

Again the residual correlation coefficients were computed. An examination of residual matrix R_b failed to reveal any additional groups. Since the residual correlations at this stage were so small, no further factors were extracted; R_b henceforth is referred to as R_1 , the first matrix of residual correlation coefficients. Matrices F_a and F_b were combined into a single factor matrix F_1 , the first matrix of orthogonal, unrotated factor loadings.

All of these computations were carried out on the IBM Card-Programmed Calculator. Since high-speed computing was available, it was decided to iterate the factor solution to stabilize both the communality estimates and the weights defining the groups. Accordingly, the communality estimates computed from factor matrix F_1 were inserted in the diagonal cells of R_0 , and the matrix F_1 was used as the new weight matrix to define the eleven groups. A new factor matrix F_2 was extracted by the method previously outlined, and the communality estimates from F_2 were computed.

The iteration of the solution to stabilize the communality estimates is a standard recommended procedure (Thurstone, 1947a, p. 295). This type of iteration of the factor weights is simply Hotelling's iterative method of factoring (Thurstone, 1947a, p. 483); this method if carried to complete convergence will yield the principal-axis factor matrix.

It was decided to iterate the solution yet another time; hence the communality estimates computed from F_2 were inserted in the diagonal cells of R_0 , and F_2 was used as the weight matrix to define the eleven groups. However, a difficulty was encountered in the process of extracting the factors. It will be noticed that (10) calls for the factoring of a small matrix T into a matrix B and its transpose B'; the diagonal or triangularization method of factoring was used (Thurstone, 1947a, pp. 101–105). In carrying out the triangularization, the lower right-hand entry in B, the last value to be computed, turned out to be an imaginary number.

This difficulty might arise from either of two causes. (i) The true rank of the reduced matrix R_0 might be ten rather than eleven. Since the solution is converging to the principal-axis solution, the amount of variance on the eleventh factor is in a sense being minimized. Hence, if the eleventh factor were actually only a random "noise" factor, one might obtain a negative root of the characteristic equation; this root could cause the imaginary number found in this analysis.

(ii) On the other hand, such an imaginary number could arise if most or all of the communality estimates were too low. It seems quite likely that this method of iterating the communality estimates might bring about this situation. Consider this problem in the following manner. Let the correlation

TABLE 7
Factor Matrix F_3:
Final Unrotated Orthogonal Factor Matrix
(Decimal points omitted)

Test	Factor											
	A_3	B_3	C_3	D_3	E_3	F_3	G_3	H_3	I_3	J_3	K_3	h_3^2
1. RSyl	37	−13	03	−01	−16	05	09	−22	08	−15	−21	32
2. RWd	32	−13	−19	−18	−06	03	40	03	23	15	−18	46
3. RFig	40	−10	23	−04	−22	08	10	06	10	−05	13	32
4. PSyl-1	45	−24	−10	−25	04	11	10	−22	−43	14	−15	63
5. PSyl-2	56	−16	−02	−32	−04	19	02	−11	−22	−06	−08	55
6. PNum	48	−20	−05	−31	−09	14	31	−09	−07	04	08	51
7. PWd-U	42	−33	−04	−10	−03	11	41	−09	02	26	16	59
8. PWd-R	50	−22	−04	−24	−13	−03	34	02	07	−20	00	54
9. SComp	53	−27	−04	−15	−16	−17	25	19	−05	−18	12	58
10. Rel	47	26	00	03	−19	17	−04	11	−08	05	−10	39
11. Con-Nv	36	−02	04	−08	−09	−12	−14	16	06	09	10	23
12. Con-V	41	−27	−06	01	−05	−20	09	19	−02	−11	−04	34
13. Lim	59	−38	00	18	−03	−33	−16	18	−10	−03	−11	71
14. Ideas	42	−31	08	08	−02	−26	−03	31	10	25	−17	55
15. NSp-Au	44	−03	−10	04	48	30	−06	08	07	05	12	56
16. NSp-Vi	49	−05	−18	−10	38	29	−13	11	07	−04	−01	55
17. LSp-Vi	48	−09	02	−09	48	20	−16	−03	12	−11	−13	59
18. LSp-Au	53	−13	−04	02	47	27	−07	−07	09	04	−12	63
19. SenSp	44	−24	−02	21	18	03	−29	−03	02	23	09	47
20. InsSp	21	01	00	−11	19	17	−26	14	−14	−24	04	29
21. Dsg-Rp	42	21	28	−02	−17	30	−07	19	−10	03	23	52
22. Map-Rp	43	15	13	−26	−19	04	08	04	06	02	00	34
23. Map-Vb	26	−00	08	−14	−04	10	09	10	04	−12	18	18
24. Map-Rc	44	13	24	−12	−17	17	02	−03	11	−04	−18	39
25. Pict	44	02	05	−08	00	−01	00	05	03	−08	−16	24
26. Para	46	−14	−08	05	−19	−11	−08	−26	23	−01	18	45
27. Num	36	11	−13	−17	−17	27	07	−09	08	07	17	35
28. IComp	40	32	21	13	06	−05	05	−03	−15	−03	−14	37
29. MPrin	32	25	27	50	04	−01	19	03	−03	09	−12	55
30. RCon	−01	14	13	23	38	−04	30	−07	−03	−10	14	36
31. CCoord	30	35	17	12	29	−12	25	02	−05	−05	07	44
32. ArReas	57	21	−25	49	−16	06	01	04	−04	−06	13	72
33. RdComp	60	−01	11	46	−10	−10	−17	−18	−03	−07	07	68
34. Vocab	61	−22	06	20	02	−27	−35	−24	−04	−03	14	74
35. DTRd	49	55	−22	−06	01	−21	−05	−03	−03	06	01	64
36. SpaO-1	35	43	18	−31	11	−25	−14	−02	03	10	24	60
37. CRd	43	53	−06	01	−08	−12	−03	01	06	08	−11	51
38. DRTime	40	29	−04	−05	11	−16	11	−04	02	04	09	31
39. SpaO-2	40	41	31	01	−10	−04	−08	−08	05	−01	−22	51
40. NumOp	46	29	−55	00	−09	−14	−12	04	−04	−08	−09	66

TABLE 8

Distribution Statistics for the Differences Between Succeeding Communality Estimates

	$h_1^2 - h_0^2$	$h_2^2 - h_1^2$	$h_3^2 - h_2^2$
Range	−.37 to .25	−.02 to .10	−.09 to .04
Mean	.026	.051	−.030
Standard deviation	.123	.024	.024

h_0^2 = original communality estimates
h_1^2 = communality estimates used in first iteration, computed from factor matrix F_1
h_2^2 = communality estimates used in second iteration, computed from factor matrix F_2
h_3^2 = communality estimates computed from factor matrix F_3

matrix, with unities in the diagonals, be factored; the complete factor matrix may be considered to be composed of three subsections—a common-factor section, a "noise" or residual-factor section, and a unique-factor section. The iteration of communality estimates in the manner described above will result in a convergence only to the values based on the common-factor section; but the correlation matrix, even when reduced by subtracting out the unique-factor section, still contains the "noise" factors, which are small to the point of being negligible but are nevertheless present. Thus it would occur that the communality estimates obtained would be underestimates of the values which actually should have been used.

Since the appearance of the imaginary number meant that if the analysis were continued the loadings on the eleventh factor would all be imaginary numbers, some action was felt to be desirable. It was decided to add .05 to each communality estimate computed from F_2 and to recompute the second iteration. If the rank of the reduced correlation matrix were truly ten instead of eleven, this fact would become apparent when the factor matrix was rotated; if the second situation outlined above prevailed, this addition of .05 should probably be enough to correct the difficulty.

After this .05 was added to the communality estimates, the solution was iterated with no further difficulty to obtain factor matrix F_3 , which is shown in Table 7; again new communality estimates were computed.

Table 8 shows the summary statistics for the frequency distributions of the differences between each set of communality estimates.

A new matrix of residual correlations, R_3 , was computed; an examination of R_3 , which is shown in Table 9, failed to reveal any additional groups.

The distribution of these residual correlation coefficients is summarized in Table 10.

TABLE 9

Final Residual Correlations: Matrix R_3

(Decimal points omitted)

Test	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1. RSyl	—	−05	10	−02	−01	−01	03	−04	02	−02	00	−01	00	−04	03	−00	−05	−00	03	−04
2. RWd	−05	—	03	−07	04	03	03	−01	01	−08	−03	03	−03	−05	−01	03	−04	−04	04	03
3. RFig	10	03	—	01	−00	02	−04	01	−03	02	01	−01	−00	−00	00	−05	01	03	03	01
4. PSyl-1	−02	−07	01	—	−01	−01	02	00	04	−04	−01	00	−03	−01	02	03	−02	−03	03	−03
5. PSyl-2	−01	04	−00	−01	—	03	−02	00	−04	−01	−02	05	−01	−00	−01	00	−01	−02	01	02
6. PNum	−01	03	02	−01	03	—	−02	01	−01	−05	−01	−02	02	−02	−02	−03	03	02	03	03
7. PWd-U	03	03	−04	02	−02	−02	—	−00	00	09	05	−02	01	01	−02	00	03	02	−05	02
8. PWd-R	−04	−01	01	00	00	01	−00	—	01	−03	00	01	−03	−01	−02	03	01	−01	04	−01
9. SComp	02	01	−03	04	−04	−01	00	01	—	−00	−05	01	04	02	03	−02	−01	05	−05	02
10. Rel	−02	−08	02	−04	−01	−05	09	−03	−00	—	05	01	−01	−01	03	00	−02	−03	−02	−02
11. Con-Nv	00	−03	01	−01	−02	−01	05	00	−05	05	—	−01	01	01	−03	01	05	−04	−00	−01
12. Con-V	−01	03	−01	00	05	−02	−02	01	01	01	−01	—	−05	−03	−00	−03	−01	−01	07	00
13. Lim	00	−03	−00	−03	−01	02	01	−03	04	−01	01	−05	—	00	−01	02	−01	03	−03	−02
14. Ideas	−04	−05	−00	−01	−00	−02	01	−01	02	−01	01	−03	00	—	03	02	−03	−03	01	−03
15. NSp-Au	03	−01	00	02	−01	−02	−02	−02	03	03	−03	−00	−01	03	—	−02	−01	04	−03	−01
16. NSp-Vi	−00	03	−05	03	00	−03	00	03	−02	00	01	−03	02	02	−02	—	00	−01	−03	01
17. LSp-Vi	−05	−04	01	−02	−01	03	03	01	−01	−02	05	−01	−01	−03	−01	00	—	−04	03	00
18. LSp-Au	−00	−04	03	−03	−02	02	02	−01	05	−03	−04	−01	03	−03	04	−01	−04	—	00	−01
19. SenSp	03	04	03	03	01	03	−05	04	−05	−02	−00	07	−03	01	−03	−03	03	00	—	01
20. InsSp	−04	03	01	−03	02	03	02	−01	02	−02	−01	00	−02	−03	−01	01	00	−01	01	—
21. Dsg-Rp	05	09	−01	03	02	−03	−06	−01	−02	02	−04	04	02	01	−04	−03	03	04	00	−00
22. Map-Rp	−03	01	01	01	−00	01	00	−02	−04	−02	04	−02	−01	04	−01	01	02	00	−08	01
23. Map-Vb	09	01	−05	05	01	−00	−08	−00	−02	04	−01	05	03	06	00	06	−01	00	−01	−06
24. Map-Rc	−03	−07	−00	−01	−02	−00	05	−03	03	02	00	−04	02	01	03	−01	02	−01	−04	−03
25. Pict	−02	−05	07	−02	−02	−04	04	−00	01	01	02	−02	−04	00	03	−02	−01	−06	02	02
26. Para	02	01	−06	03	00	−03	−01	−01	−01	04	−03	04	02	03	−02	01	03	02	−04	01
27. Num	04	02	−09	02	−01	−04	−05	02	−01	06	00	−04	01	06	−02	−01	−01	01	−00	−03
28. IComp	−00	−02	02	−03	−03	−01	07	02	03	−03	−00	−03	02	−05	01	−01	−02	−01	04	04
29. MPrin	−09	−02	02	−01	01	05	00	03	01	−08	−05	01	−02	−02	00	02	−02	−03	04	04
30. RCon	02	04	−02	02	02	−07	−00	−01	−01	03	03	00	−01	02	−04	−00	03	−01	01	−01
31. CCoord	07	−02	−03	−01	−01	04	−03	−02	−07	06	02	−01	04	06	−00	−00	01	01	−06	−05
32. ArReas	00	05	−03	04	02	01	−05	01	−02	−01	01	01	01	02	−04	03	03	02	−00	00
33. RdComp	01	03	−03	−01	01	−03	01	−00	02	02	00	01	−01	01	00	00	−01	01	−02	02
34. Vocab	03	05	−02	00	01	−01	−03	−00	−03	02	−01	−02	02	02	01	−00	00	00	−02	−00
35. DTRd	02	03	02	02	00	−02	−02	−03	01	04	−02	02	−00	−00	−00	−03	−01	04	00	−00
36. SpaO-1	01	06	−05	04	03	−03	−05	03	−01	00	−03	03	00	03	−05	03	−01	04	02	03
37. CRd	02	−02	03	01	−01	05	00	−01	06	−06	−03	00	−00	−03	04	01	−02	−00	03	−00
38. DRTime	00	04	01	−00	−01	−01	−02	−02	−01	04	02	01	−00	−02	03	−04	04	−02	−03	−01
39. SpaO-2	−08	−03	−02	−02	−01	03	03	03	04	−06	02	−01	−03	−05	01	03	−02	−03	04	−01
40. NumOp	−01	−05	02	−02	−02	02	03	01	−01	−03	02	−03	00	−00	01	−01	00	−02	02	−00

TABLE 9 (cont.)
Final Residual Correlations: Matrix R_3
(Decimal points omitted)

Test	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1. RSyl	05	−03	09	−03	−02	02	04	−00	−09	02	07	00	01	03	02	01	02	00	−08	−01
2. RWd	09	01	01	−07	−05	01	02	−02	−02	04	−02	05	03	05	03	06	−02	04	−03	−05
3. RFig	−01	01	−05	−00	07	−06	−09	02	02	−02	−03	−03	−03	−02	02	−05	03	01	−02	02
4. PSyl-1	03	01	05	−01	−02	03	02	−03	−01	02	−01	04	−01	00	02	04	−01	−00	−02	−02
5. PSyl-2	02	−00	01	−02	−02	00	−01	−03	01	02	−01	02	01	01	00	03	01	−01	−01	−02
6. PNum	−03	01	−00	−00	−04	−03	−04	−01	05	−07	04	01	−03	−01	−02	−03	05	−01	03	02
7. PWd-U	−06	00	−08	05	04	−01	−05	07	00	−00	−03	−05	01	−03	−02	−05	00	−02	03	03
8. PWd-R	−01	−02	−00	−03	−00	−01	02	02	03	−01	−02	01	−00	−00	−03	03	−01	−02	03	01
9. SComp	−02	−04	−02	03	01	−01	−01	03	01	−01	−07	−02	02	−03	01	−01	06	−01	04	−01
10. Rel	02	−02	04	02	01	04	06	−03	−08	03	06	−01	02	02	04	00	−06	04	−06	−03
11. Con-Nv	−04	04	−01	00	02	−03	00	−00	−05	03	02	01	00	−01	−02	−03	−03	02	02	02
12. Con-V	04	−02	05	−04	−02	04	−04	−03	01	00	−01	01	01	−02	02	03	00	01	−01	−03
13. Lim	02	−01	03	02	−04	02	01	02	−02	−01	04	01	−01	02	−00	00	−00	−00	−03	00
14. Ideas	01	04	06	01	00	03	06	−05	−02	02	06	02	01	02	−00	03	−03	−02	−05	−00
15. NSp-Au	−04	−01	00	03	03	−02	−02	01	00	−04	−00	−04	00	01	−00	−05	04	03	01	01
16. NSp-Vi	−03	01	06	−01	−02	01	−01	−01	02	−00	−00	03	00	−00	−03	03	01	−04	03	−01
17. LSp-Vi	03	02	−01	02	−01	03	−01	−02	−02	03	01	03	−01	00	−01	−01	−02	04	−02	00
18. LSp-Au	04	00	00	−01	−06	02	01	−01	−03	−01	01	02	01	00	04	04	−00	−02	−03	−02
19. SenSp	00	−08	−01	−04	02	−04	−00	04	04	01	−06	−00	−02	−02	00	02	03	−03	04	02
20. InsSp	−00	01	−06	−03	02	01	−03	04	04	−01	−05	00	02	−00	−00	03	−00	−01	−01	−00
21. Dsg-Rp	—	00	−07	−02	05	−05	00	04	04	−04	−01	−07	−02	−00	−02	−07	03	01	05	04
22. Map-Rp	00	—	01	−01	−01	02	−00	−03	02	02	−00	03	−01	04	−00	−00	−05	−01	02	02
23. Map-Vb	−07	01	—	06	−07	−04	00	00	−01	02	−00	−01	−01	−03	02	−02	02	−06	06	−01
24. Map-Rc	−02	−01	06	—	01	02	04	−02	−02	01	01	01	01	00	02	02	00	−02	−04	−01
25. Pict	05	−01	−07	01	—	04	09	−00	−04	03	05	−02	−01	03	00	−01	−05	06	−08	02
26. Para	−05	02	−04	02	04	—	01	−00	05	−02	−01	−04	−02	−03	−01	−05	01	−00	06	01
27. Num	00	−00	00	04	09	01	—	−01	−01	03	02	−04	02	−03	−04	−01	−06	03	00	03
28. IComp	04	−03	00	−02	−00	−00	−01	—	−05	01	−03	01	01	−00	03	01	04	02	−03	−03
29. MPrin	04	02	−01	−02	−04	05	−01	−05	—	02	−02	06	00	00	−02	07	02	01	−01	−01
30. RCon	−04	02	02	01	03	−02	03	01	02	—	−00	−03	−01	00	02	−02	−01	−07	00	02
31. CCoord	−01	−00	−00	01	05	−01	02	−03	−02	−00	—	−03	−02	−00	−04	−05	−05	04	03	04
32. ArReas	−07	03	−01	01	−02	−04	−04	01	06	−03	−03	—	−01	−03	−02	−03	00	−03	09	02
33. RdComp	−02	−01	−01	01	−01	−02	02	01	00	−01	−02	−01	—	00	02	01	−02	−00	00	−02
34. Vocab	−00	04	−03	00	03	−03	−03	−00	00	00	−00	−03	00	—	−02	−04	00	02	01	02
35. DTRd	−02	−00	02	02	00	−01	−04	03	−02	02	−04	−02	02	−02	—	−00	04	−04	−03	−01
36. SpaO-1	−07	−00	−02	02	−01	−05	−01	01	07	−02	−05	−03	01	−04	−00	—	02	−01	09	01
37. CRd	03	−05	02	00	−05	01	−06	04	02	−01	−05	00	−02	00	04	02	—	−05	−03	−02
38. DRTime	01	−01	−06	−02	06	−00	03	02	01	−07	04	−03	−00	02	−04	−01	−05	—	−01	01
39. SpaO-2	05	02	06	−04	−08	06	00	−03	−01	00	03	09	00	01	−03	09	−03	−01	—	−02
40. NumOp	04	02	−01	−01	02	01	03	−03	−01	02	04	02	−02	02	−01	01	−02	01	−02	—

TABLE 10

Distribution Statistics for Residual Correlations in Matrix R_3

Range	−.0890 to .0979
Mean	−.0005
Standard deviation	.0300

Since the factor matrix was to be rotated to a psychologically meaningful position, it was not necessary to obtain complete convergence to the principal-axis solution. At this time the solution was near enough to the principal space, so no further iterations were performed; thus F_3 (Table 7) is the final unrotated orthogonal factor matrix, and R_3 (Table 9) is the final residual correlation matrix.

Rotation of Axes

The rotation of axes was greatly facilitated by the use of the Matrix Rotator at The Adjutant General's Office in Washington, D. C. First, the axes were rotated orthogonally until a fairly good approximation to simple structure was obtained; then the axes were rotated obliquely to improve the simple-structure approximation. Nine additional graphic rotations were later made in order to clarify the structure further. The results of these rotations are shown in Tables 11–14. Table 11 presents the oblique transformation matrix Λ, by which the orthogonal factor matrix F_3, shown in Table 7, is transformed into the rotated oblique factor matrix V, shown in

TABLE 11

Oblique Transformation Matrix Λ
(Decimal points omitted)

Factor	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
A_3	22	18	26	30	25	20	08	10	25	13	20
B_3	−27	43	67	−21	−33	−07	−04	−18	28	19	02
C_3	02	−81	17	−22	11	−04	02	01	51	14	06
D_3	32	23	−46	−30	−17	−14	−17	11	28	63	03
E_3	−12	−11	01	09	−11	73	04	09	−44	33	−28
F_3	−17	05	−39	35	−34	40	05	−21	47	−08	56
G_3	−29	−02	−06	53	14	−34	−07	−05	01	57	04
H_3	−60	20	09	−33	54	10	−25	53	02	−03	48
I_3	25	−12	11	19	00	13	−91	−02	03	−27	−19
J_3	13	−01	24	41	−56	−27	15	76	−11	−04	21
K_3	46	−01	07	03	−18	−16	−19	−17	−33	12	51

TABLE 12

Rotated Oblique Factor Matrix *V*
(Decimal points omitted)

Test	Factor										
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
1. RSyl	13	−05	−07	21	16	01	03	−16	24	−02	−11
2. RWd	−09	09	06	53	15	−04	−13	16	−02	−01	00
3. RFig	14	−14	06	14	21	−07	−11	−02	28	00	26
4. PSyl-1	00	01	−03	41	03	09	58	06	−02	−00	06
5. PSyl-2	−00	−01	06	34	17	20	35	−08	12	−12	15
6. PNum	03	−03	05	53	15	−00	16	−03	02	−01	21
7. PWd-U	19	−08	−08	64	02	−10	05	17	−05	14	25
8. PWd-R	−01	−03	04	36	43	−00	−04	−10	04	01	07
9. SComp	02	03	05	19	53	−08	−01	04	−03	05	18
10. Rel	−07	26	22	07	03	01	10	07	39	03	28
11. Con-Nv	11	03	23	01	16	−00	−06	19	03	−11	16
12. Con-V	03	05	−03	05	44	01	−02	15	−04	05	03
13. Lim	19	02	−06	−10	48	06	09	32	01	05	−04
14. Ideas	06	−06	06	05	35	−01	−08	54	00	−01	03
15. NSp-Au	05	12	−01	27	−06	57	−03	10	−05	16	21
16. NSp-Vi	−03	18	01	26	06	59	−00	05	−03	−03	16
17. LSp-Vi	03	−04	02	20	09	66	01	−01	02	−00	−06
18. LSp-Au	09	02	−05	33	−02	61	05	08	04	11	01
19. SenSp	38	01	−08	10	−08	24	04	28	−03	03	12
20. InsSp	−10	05	00	−14	16	41	12	−11	03	−09	14
21. Dsg-Rp	−00	02	22	02	02	04	05	02	43	05	55
22. Map-Rp	−04	−00	35	19	15	−04	−00	−01	25	−10	18
23. Map-Vb	−01	−03	10	11	18	07	−07	−09	09	−00	24
24. Map-Rc	−03	−08	21	16	11	08	−00	−06	46	−07	12
25. Pict	−04	03	16	09	22	15	03	02	18	−01	00
26. Para	51	04	05	20	02	−07	−16	−10	03	−08	−00
27. Num	11	18	14	36	−11	01	−02	−11	15	−10	31
28. IComp	−05	07	28	−06	04	04	16	00	32	30	−00
29. MPrin	04	06	07	−04	−04	−08	−02	15	44	55	07
30. RCon	−01	−06	−01	03	−06	11	−05	−11	−05	52	−08
31. CCoord	−07	06	31	03	03	09	−00	−02	09	48	−01
32. ArReas	25	54	−01	01	−01	−06	−06	−00	28	36	28
33. RdComp	48	09	−03	−09	04	−02	02	−02	31	28	05
34. Vocab	60	−04	03	−06	10	08	10	03	−01	03	−08
35. DTRd	00	47	58	03	−06	−01	05	02	03	10	−02
36. SpaO-1	08	−01	70	−02	−04	04	02	02	−04	−05	04
37. CRd	−04	35	52	00	−05	−04	−02	06	24	08	00
38. DRTime	06	18	39	14	00	02	−01	02	−01	18	00
39. SpaO-2	−02	−01	43	−07	01	01	03	−04	48	07	−05
40. NumOp	−00	67	26	02	08	05	04	−01	−07	−04	−03

TABLE 13

Cosines of Angles Between Reference Vectors: Matrix $\Lambda'\Lambda$
(Decimal points omitted except on principal diagonal)

Factor	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
A	1.00	−16	−15	11	−36	−18	−16	−16	−12	−01	−10
B	−16	1.00	07	−02	−12	−04	00	04	−11	11	17
C	−15	07	1.00	−04	−03	−12	−04	16	−02	−14	−05
D	11	−02	−04	1.00	−30	03	03	07	−11	03	13
E	−36	−12	−03	−30	1.00	06	−15	01	00	−07	−06
F	−18	−04	−12	03	06	1.00	−04	−11	−08	−11	−08
G	−16	00	−04	03	−15	−04	1.00	03	−01	09	01
H	−16	04	16	07	01	−11	03	1.00	−15	01	21
I	−12	−11	−02	−11	00	−08	−01	−15	1.02	11	29
J	−01	11	−14	03	−07	−11	09	01	11	1.00	03
K	−10	17	−05	13	−06	−08	01	21	29	03	1.01

Table 12. Table 13 presents the cosines of the angles between the reference vectors, while Table 14 contains the correlations between the primary factors.

Throughout the report the entries in the V matrix will be called "factor coefficients" rather than "factor loadings." In the oblique case, a factor loading is generally considered to be the oblique projection of a test vector on a primary-factor vector; an entry in the V matrix, however, represents the orthogonal projection of a test vector on a reference-factor vector.

TABLE 14

Intercorrelations of Primary Vectors: Matrix TT'
(Decimal points omitted except on principal diagonal)

Factor	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
A	1.00	25	19	02	43	26	28	18	20	02	02
B	25	1.00	−05	11	21	07	09	09	24	−13	−22
C	19	−05	1.00	03	13	16	09	−14	−01	17	12
D	02	11	03	1.00	28	−04	03	−02	16	−04	−17
E	43	21	13	28	1.00	06	25	03	11	03	00
F	26	07	16	−04	06	1.00	09	13	11	11	04
G	28	09	09	03	25	09	1.00	02	08	−07	00
H	18	09	−14	−02	03	13	02	1.00	25	−04	−26
I	20	24	−01	16	11	11	08	25	1.00	−13	−36
J	02	−13	17	−04	03	11	−07	−04	−13	1.00	06
K	02	−22	12	−17	00	04	00	−26	−36	06	1.00

The exact relationships between all the variables in the oblique case have been shown (Thurstone, 1947a, pp. 347–359); among other relations it was shown that the factor coefficients are proportional to the factor loadings. Some of these relationships may be expressed in the following manner.

R_{ik} = reduced correlation matrix.
F_{im} = unrotated orthogonal factor matrix.
Λ_{mo} = oblique transformation (rotation) matrix.
V_{io} = rotated oblique matrix of factor coefficients.
D_{po} = orthogonal projections of primary-factor vectors on reference-factor vectors. (This is a diagonal matrix; all off-diagonal cell entries are zero.)
A_{ip} = rotated oblique matrix of factor loadings.

Then

$$R_{ik} = F_{im}F'_{im} . \tag{13}$$

$$F_{im}\Lambda_{mo} = V_{io} . \tag{14}$$

$$V_{io}D_{po}^{-1} = A_{ip} . \tag{15}$$

$$R_{ik} = V_{io}(\Lambda'_{mo}\Lambda_{mo})^{-1}V'_{io} . \tag{16}$$

$$R_{ik} = A_{ip}R_{pq}A'_{jp} , \quad \text{where} \quad R_{pq} = D_{po}(\Lambda'_{mo}\Lambda_{mo})^{-1}D_{po} . \tag{17}$$

The V matrix of factor coefficients is the matrix usually reported in factor analysis studies using oblique rotations; quite commonly, however, the entries in this matrix are referred to as factor loadings. The term "factor coefficients" has been adopted in order to indicate precisely that the V matrix, not the A matrix, is being reported.

In interpreting the factors, either matrix (coefficients or loadings) will place the tests in the same rank order of magnitude; in general, the magnitude of the factor coefficients seem to the author to be more useful in making the interpretation. Since in most other obliquely rotated analyses the V matrix has been reported instead of the A matrix, it seems logical to assume that other authors have also found it preferable. The interpretation of the factors will be presented in the following chapter.

Chapter IV

RESULTS AND INTERPRETATION

The results of the factor analysis of the battery of 40 tests, together with the interpretation of these results, are given below. The rotated oblique factor matrix, V, was presented in Table 12, on page 31. The eleven factors are interpreted in the following pages; the information in Tables 15–25 was taken from Table 12.

Factor A: Verbal Comprehension

This factor appears to represent knowledge of language and facility in the manipulation of verbal material; it corresponds to Factor V as identified by French (1951, p. 244). The interpretation of this factor seems to be quite clear.

Factor B: Numerical Facility

This factor seems to represent facility in the manipulation of numbers; it corresponds to Factor N as identified by French (1951, p. 225). Again, there does not seem to be much question as to the interpretation of this factor.

One might ask about the two number-span tests, tests 15 and 16, and about the paired-associates word-number test, test 6. The two number-span tests have factor coefficients in the indeterminate range where it is not at all clear whether to consider the coefficients as being significantly different

TABLE 15

Tests High on Factor A

Test	Factor Coefficient
*34. Vocabulary	.60
*26. Meaningful Memory: Paragraph	.51
*33. Reading Comprehension	.48
*19. Sentence Span	.38
32. Arithmetic Reasoning	.25

*This test had its highest factor coefficient on this factor.

TABLE 16

Tests High on Factor *B*

Test	Factor Coefficient
*40. Numerical Operations	.67
*32. Arithmetic Reasoning	.54
35. Dial and Table Reading	.47
37. Coordinate Reading	.35
10. Memory for Relations	.26

*This test had its highest factor coefficient on this factor.

from zero; the coefficient of the word-number test is clearly not significantly different from zero. The magnitudes of these coefficients fit in with the interpretation of this factor made above; i.e., the mere presence of numbers in a test is not sufficient for that test to have a high coefficient on this factor. Rather, the tests with high coefficients on this factor all seem to require the manipulation of numbers in some manner.

Factor C: Perceptual Speed

This factor appears to represent the ability to perceive rapidly; tests with high coefficients involve the rapid inspection and comparison of visual forms, with the notation of similarities and differences in form and detail. This factor corresponds to Factor *P* as identified by French (1951, p. 277). The identification of this factor also seems quite clear.

Factor D: Rote Memory

There seems to be little doubt that this is a memory factor; it is interpreted as representing the ability to remember bits of unrelated factual material. This factor corresponds to Factor *M* identified by French (1951, p. 219).

Tests 1–7, inclusive, were constructed in an attempt to measure the hypothesized factor of Rote Memory; all of these tests, except test 3, have high coefficients on this Factor *D*. In addition, tests 2, 6, and 7 have their highest coefficients on this factor, while tests 4 and 5 have higher coefficients only on a doublet factor which represents the common variance of these two paired-associates nonsense-syllable tests. Test 1, while being low on this factor, is low on all factors, its highest coefficient being only .24 on Factor *I*. Since the communality is only .32, either the test is very unreliable or it has a great amount of specific variance.

Now, what is the nature of tests 8, 15, 16, 17, 18, 26, and 27, the tests with moderately high coefficients on this factor, which were designed to be

TABLE 17

Tests High on Factor *C*

Test	Factor Coefficient
*36. Spatial Orientation I	.70
*35. Dial and Table Reading	.58
*37. Coordinate Reading	.52
39. Spatial Orientation II	.43
*38. Discrimination Reaction Time	.39
*22. Map Memory I (Reproduction)	.35
31. Complex Coordination	.31
28. Instrument Comprehension	.28
40. Numerical Operations	.26
*11. Consequences I (Nonverbal)	.23
21. Reproduction of Visual Designs	.22
10. Memory for Relations	.22
24. Map Memory III (Recognition)	.21

*This test had its highest factor coefficient on this factor.

primarily measures of other factors? Tests 15, 16, 17, and 18 are the number-span and letter-span tests; it would seem reasonable for these tests to have high coefficients on a rote memory factor. However, their coefficients on Factor *D* are comparatively low, and they all have higher coefficients on Factor *F*; more will be said about them when Factor *F* is discussed. Tests 26 and 27 were two of the three experimental tests not specifically constructed for this study, but it was thought that they would be primarily measures of a meaningful memory factor; more will be said about these two tests in the discussion of Factor *E* below. Test 8 will also be discussed with Factor *E*. However, in examining the nature of each of these tests, it does not seem particularly surprising that they should appear on a rote memory factor as defined above.

Most of the tests with high coefficients on this factor are composed of relatively discrete elements; in general, no obvious relationships exist among the elements. There are a few tests where some relationships do exist, tests 8, 26, and 27. (Test 8 had a higher coefficient on Factor *E*, to be discussed next.) It seems not unlikely that on these three tests some examinees did not make use of the relationships in remembering the elements, which would account for the appearance of these tests on this Rote Memory factor. This possibility could be checked by getting introspective reports from examinees.

Consider now test 3, which was also constructed to be a measure of Rote Memory. This test requires the recognition of previously studied geometric figures or symbols. This test had its highest coefficient on Factor *I* (.28), with

TABLE 18

Tests High on Factor *D*

Test	Factor Coefficient
*7. Memory for Words I (Unrelated)	.64
*2. Recognition II (Words)	.53
*6. Memory for Numbers	.53
4. Memory for Syllables I	.41
*27. Meaningful Memory: Number	.36
8. Memory for Words II (Related)	.36
5. Memory for Syllables II	.34
18. Letter Span II (Auditory)	.33
15. Number Span I (Auditory)	.27
16. Number Span II (Visual)	.26
1. Recognition I (Syllables)	.21
17. Letter Span I (Visual)	.20
26. Meaningful Memory: Paragraph	.20

*This test had its highest factor coefficient on this factor.

additional coefficients of moderate size in Factors *K* (.26) and *E* (.21); its coefficient on Factor *D* was only .14. As will be indicated in the discussions of Factors *E*, *I*, and *K*, this test seems to be more closely related to the visualization and visual memory tests than it does to the rote memory tests. A re-examination of the figures in this test showed that a number of them appear to be representations of objects; perhaps this accounts for its relation to meaningful memory tests. In any case, test 3 had a communality of only .32, indicating either unreliability or a high proportion of specificity.

What conclusions can be drawn about this factor? First, the Rote Memory factor did not split into two separate factors—recognition and paired associates. The memory tests in this battery may be roughly classified into three categories: Recognition, Completion Recall (including paired associates and multiple choice), and Free Recall. It may be seen in Table 18 that tests from all three of the categories appear on this factor; it is true that the five paired-associates tests are among the top seven tests, while one of the other two tests in this top group is also a completion-recall test. However, test 2, with the second highest coefficient on this factor is a recognition test; and the number-span and letter-span tests, with lower coefficients, are free-recall tests. Thus on the basis of this study it may be concluded that completion-recall tests, in particular paired-associates tests, are the best measures of this factor, but that the factor is not restricted to tests of this kind.

It can be seen that the content of the tests with high coefficients on the Rote Memory factor includes numbers, letters, nonsense syllables, and words.

TABLE 19
Tests High on Factor *E*

Test	Factor Coefficient
*9. Sentence Completion	.53
*13. Memory for Limericks	.48
*12. Consequences II (Verbal)	.44
*8. Memory for Words II (Related)	.43
14. Memory for Ideas	.35
*25. Meaningful Memory: Picture	.22
3. Recognition III (Figures)	.21

*This test had its highest factor coefficient on this factor.

This factor is not specific to one modality of presentation, since some of the tests were presented visually and some were presented auditorily. However, all of this material is verbal, and the testing necessitates verbalization of the material by the examinee; hence it is quite possible that this factor is restricted to verbal material. (The absence of a high coefficient for test 3 lends support to this conjecture.) This is a problem which will require further investigation.

Factor E: Meaningful Memory

This factor seems to represent the ability to remember material which is meaningful. The term "meaningful" has not been precisely defined; only an intuitive definition of its meaning has been implied, both in the test construction and in the identification made above.

Tests 8–14, inclusive, were specifically constructed in an attempt to measure the hypothesized factor of "Meaningful Memory," while tests 26 and 27 were also thought to be measures of such an ability. It will be noted that five of the seven specifically constructed tests (numbers 10 and 11 are the exceptions) are high on this factor; of these five, only test 14 does not have its highest coefficient here.

Several specific tests need to be considered. First, what happened to tests 10 and 11? Test 10, Memory for Relations, had essentially a zero coefficent on this factor; although this test seems to line up more with the visual tests than with those on this factor, its nature is not made clear in the analysis of this test battery. The coefficients on Factors *I* and *K* perhaps indicate that the examinees were not making use of the relations in remembering the matrices; the relations themselves may have been too difficult to be discovered in the time allowed for this test. If this were true, it would not be surprising to find that those examinees who are high on a visual memory ability make the better scores on this test.

Test 11, Consequences I (Nonverbal), turned out to be much too easy for this population; the score distribution was extremely negatively skewed, with the large majority of examinees making perfect or near-perfect scores; its coefficient on this factor was in the indeterminate range of possible significance.

Next, what about tests 26 and 27, which did not appear on this factor either? Test 26 seems to be primarily a measure of verbal comprehension, with a slight component of Rote Memory, while test 27 seems to be fairly complex, with a moderate coefficient on the Rote Memory factor. If this factor is indeed Meaningful Memory, then it might be suggested that either there is not enough "meaning" in tests 26 and 27 for them to appear on this factor, or else that the type of "meaning" they contain is not that which distinguishes this factor. In addition, test 27 was too easy for this group, thus giving a very skewed score distribution.

Finally, consider tests 3 and 25, which appear on this factor with comparatively low coefficients. As previously mentioned, an examination of the nature of test 3, Recognition III (Figures), showed that a number of the figures seem to be representations of objects; perhaps this might explain its coefficient on this factor; however, this test also has no very high coefficients, so its nature remains questionable. In test 25, Meaningful Memory: Picture, there is, at least in some sense, meaning. However, this coefficient of .22 is the highest coefficient that this test has on any factor, so perhaps the best that can be said is that the nature of this test is not clarified by this battery.

The following conclusions with regard to Factor *E* may now be stated. The modality of presentation of the test material is not a discriminatory characteristic for this factor; both visually presented and auditorily presented tests have high coefficients.

All three types of tests—recognition, completion recall, and free recall—appear on this factor. Although the free-recall and recognition tests have low coefficients, it seems justified to conclude on the basis of these results that this factor is not restricted to completion-recall tests.

With regard to the content of the tests the evidence is somewhat uncertain. The tests with the highest coefficients are all composed of verbal material (words and sentences), but the tests with lower coefficients involve nonverbal material (pictures and geometric figures). It is unfortunate that test 11, Consequences (Nonverbal), turned out to be too easy, since it was intended to clarify this very point, i.e., whether or not this factor is limited to verbal material. It is necessary to conclude that this question is not completely settled by this study; however, the results seem to indicate that this factor is not limited to verbal material.

Taking all this information, along with the nature of the tests which have essentially zero coefficients on this factor, into consideration, some element of

TABLE 20

Tests High on Factor *F*

Test	Factor Coefficient
*17. Letter Span I (Visual)	.66
*18. Letter Span II (Auditory)	.61
*16. Number Span II (Visual)	.59
*15. Number Span I (Auditory)	.57
*20. Memory for Instructions	.41
19. Sentence Span	.24
5. Memory for Syllables II	.20

*This test had its highest factor coefficient on this factor.

meaning seems to be the characteristic which distinguishes between those tests which have high coefficients and those which have low coefficients.

Factor F: Span Memory

As hypothesized, this factor seems to represent the ability to recall perfectly for immediate reproduction a series of unrelated items after only one presentation of the series.

Tests 15–20, inclusive, were constructed in an attempt to measure Span Memory; it can be seen that all of these tests are present on this factor, and all but one of them has its highest coefficient on this factor. Test 19, Sentence Span, has its highest coefficient on the Verbal factor, a reasonable result. The only other test in the battery which appears on this Span Memory factor is test 5, Memory for Syllables II. Tests 4 and 5, which were designed to be parallel tests, were both too difficult for this population, and from the factorial results it seems logical to assume that the examinees changed their method of learning the syllables between the first and the second tests.

The tests in this study which had high coefficients on this Span Memory

TABLE 21

Tests High on Factor *G*

Test	Factor Coefficient
*4. Memory for Syllables I	.58
*5. Memory for Syllables II	.35

*This test had its highest factor coefficient on this factor.

TABLE 22
Tests High on Factor *H*

Test	Factor Coefficient
*14. Memory for Ideas	.54
13. Memory for Limericks	.32
19. Sentence Span	.28

*This test had its highest factor coefficient on this factor.

factor were all free-recall tests, with the exception of test 5, a completion-recall test, which was discussed above. It is quite probable that the Span Memory factor is restricted to tests of free recall; this should be investigated further.

With regard to content, these span tests included numbers, letters, words, and sentences—all verbal material. It would be highly desirable to determine whether tests of nonverbal material are also measures of this factor.

This study presents fairly clear evidence that visually presented and auditorily presented span tests measure the same factor rather than two different factors as might have been the case.

Factor G: Doublet

This factor quite clearly represents only the specific variance in tests consisting of nonsense syllables presented in the form of paired associates. It cannot be considered to be a group factor for paired associates tests in general, since none of the other tests of this kind has a significant coefficient on this factor. Thus it can be concluded that a test of nonsense-syllable paired associates measures something distinct from everything that is measured by all the other tests in the battery, but there is not enough information here to indicate the exact nature of this specific ability or abilities.

Factor H: Unidentified Triplet

There are not enough tests with high coefficients on this factor for any clear interpretation of its nature. Perhaps the most logical guess is that this may be an Auditory Memory factor, representing the ability to remember material by the formation of an auditory image. Both tests 14 and 19 were presented auditorily; while test 13 was presented visually, the type of material composing this test would lend itself quite well to auditory retention as defined above. Three other tests which were presented auditorily—test 2, Recognition II (Words); test 7, Memory for Words I (Unrelated); and test 12, Consequences II (Verbal)—have coefficients in the indeterminate range of magnitude, but two visually presented tests—test 11, Consequences I

TABLE 23

Tests High on Factor *I*

Test	Factor Coefficient
*39. Spatial Orientation II	.48
*24. Map Memory III (Recognition)	.46
29. Mechanical Principles	.44
21. Reproduction of Visual Designs	.43
*10. Memory for Relations	.39
*28. Instrument Comprehension	.32
33. Reading Comprehension	.31
*3. Recognition III (Figures)	.28
32. Arithmetic Reasoning	.28
22. Map Memory I (Reproduction)	.25
*1. Recognition I (Syllables)	.24
37. Coordinate Reading	.24

*This test had its highest factor coefficient on this factor.

(Nonverbal), and test 29, Mechanical Principles—also have coefficients in this range.

It must be noted, however, that none of the auditorily presented span tests, with the exception of Sentence Span, appears on this factor, so it is not an auditory-presentation factor. Perhaps it would be more precise to think of it as an Auditory Verbal Memory factor. This factor is not clearly identifiable with any of the factors reported by Karlin (1941, 1942).

It is also possible that this triplet represents another kind of meaningful memory factor, although it is not at all clear what the nature of such a factor is.

Factor I: Visualization?

As will be seen in the consideration of Factors *J* and *K* below, this battery of tests was not so chosen as to indicate clearly the structure in the visual-spatial factor domain; Factors *I*, *J*, and *K* seem to represent the variance which has been attributed in previous analyses (French, 1951; Guilford, 1947) to factors of Visualization, Space, Deduction, Psychomotor Coordination, and perhaps Visual Memory as well. This confusion most likely is due to the structure of the test battery; unfortunately, it seems that the reference tests did not include enough relatively independent measures of each of the factors mentioned above to separate them clearly in the analysis. The confounding resulted from the complex interrelations of the reference tests used. For this reason, the identifications made for the following three factors are all very tentative and not at all clear-cut.

TABLE 24

Tests High on Factor *J*

Test	Factor Coefficient
*29. Mechanical Principles	.55
*30. Rudder Control	.52
*31. Complex Coordination	.48
32. Arithmetic Reasoning	.36
28. Instrument Comprehension	.30
33. Reading Comprehension	.28

*This test had its highest factor coefficient on this factor.

This factor seems most nearly to correspond to the Visualization factor reported by the Air Force (Guilford, 1947). This factor has been defined by French (1951, p. 247) as the ability to comprehend imaginary movements in three-dimensional space, or the ability to manipulate objects in the imagination. Many of the tests with high coefficients on this factor, however, do not involve any such manipulation, so this definition does not seem to be very good for Factor *I*; therefore, this identification is made only very tentatively. There is some indication that this factor may represent a confounding of Visualization and Visual Memory.

Factor J: Kinesthetic-Spatial-Reasoning?

This factor seems to represent a confounding of three of the factors defined by French (1951)—Psychomotor Coordination, Space, and Deduction. French interpreted Psychomotor Coordination as representing the ability either to integrate muscular movements or to coordinate the eye and muscular movements, especially of the hand. The Space factor was interpreted as the ability to perceive spatial patterns accurately and to compare them with each other; the Space factor may or may not be limited to Visual perception. The Deduction factor was identified as the ability to reason from the general to the specific. This identification of Factor *J* is very tentative.

Factor K: Unidentified

It will be noticed that only one test has a very high coefficient on this factor; the other tests which seem to be on this factor have only low to moderate coefficients. In this situation the position of the hyperplane was not too clearly defined, and there does not seem to be any interpretation which is clear enough to warrant giving this factor any identifying title at all. It seems likely that if the nature of Factors *I* and *J* were clarified, this factor

TABLE 25

Tests High on Factor *K*

Test	Factor Coefficient
*21. Reproduction of Visual Designs	.55
27. Meaningful Memory: Number	.31
10. Memory for Relations	.28
32. Arithmetic Reasoning	.28
3. Recognition III (Figures)	.26
7. Memory for Words I (Unrelated)	.25
*23. Map Memory II (Verbal)	.24
15. Number Span I (Auditory)	.21
6. Memory for Numbers	.21

*This test had its highest factor coefficient on this factor.

might also be clarified; in that case it is possible that this factor might well turn out to represent the hypothesized factor of Visual Memory. From this analysis of the battery as it was constituted, it is not possible to conclude that a Visual Memory factor exists; however, neither is it possible to cite these results as evidence that such a factor does not exist, since on this point the results are too equivocal.

In spite of the equivocality of the results it still seems likely that a Visual Memory factor could be demonstrated in another study. Clearly, there should be more and better tests constructed which are specially designed to measure such a factor, and the reference tests in the battery need to be very carefully chosen in order to avoid the confusion in factor structure encountered in this study.

General Comments

The intercorrelations of these primary factors were shown in Table 14 on page 32. It can be seen that the intercorrelations of Factors *A* through *H* have in general either approximately zero or low positive values. The only correlation coefficient in this group which exceeds .28 is that of .43 between Factor *A* (Verbal Comprehension) and Factor *E* (Meaningful Memory). This particular correlation is not surprising since, as has already been pointed out, the tests with the highest factor coefficients on Meaningful Memory were all composed of verbal material; if better nonverbal tests of Meaningful Memory can be developed, it is to be expected that this correlation will decrease. The other correlations seem to be reasonable in terms of the restriction of the population. It may be noted that the Rote and Meaningful Memory factors correlate only .28, while Span Memory fails to correlate with either of these other two

memory factors; thus there seems to be no evidence for a general second-order memory factor. Since Factors *I*, *J*, and *K* are so poorly determined, no particular significance can be attached to their correlations either with the other factors or among themselves; however, it may be noted that none of these correlations is especially high.

In summary, it was possible to give clear interpretations for three reference factors, three memory factors, and one doublet factor; three factors involving visual tasks remain unclear; and the other factor, a triplet, is suggestive but uncertain. The following chapter will summarize the entire study, and recommendations for further research will be made.

Chapter V

SUMMARY AND RECOMMENDATIONS

Summary of Study

This study was undertaken to investigate the factorial structure in that part of the area of memory involving relatively immediate intentional retention. The development of 24 tests of memory was guided by the following factors which were hypothesized as covering important parts of this area of the memory domain.

1. Rote Memory: the ability to recall learned, meaningless material.
2. Meaningful Memory: the ability to recall learned, meaningful material.
3. Span Memory: the ability to recall perfectly a series of unrelated items after only one presentation of the series.
4. Visual Memory: the ability to recall material learned by the formation of an image of a whole visual field.

Three additional memory tests previously constructed by the Educational Testing Service were also included in this study.

In order to assess the generality of whatever memory factors might be found, tests of varied content and type were used. The battery included both verbal and nonverbal tests; some tests required recognition, some depended upon completion recall, while others depended upon free recall; some tests were presented visually, while others were administered auditorily. The material used included numbers, letters, nonsense syllables, words, sentences, stories, limericks, maps, pictures, and geometric designs and symbols. A few tests of delayed retention were included, but the longest delay was only about 30 minutes.

Since the factor analysis was operationally independent of the hypotheses, once the test battery was constructed, the analysis reveals the structure underlying the behaviors covered by the test battery; thus the analysis provides a check on the foregoing hypotheses or indicates alternative hypotheses. To clarify the nature of the memory factors, 13 reference tests measuring previously identified aptitude factors were included in the battery, bringing the total number of tests up to 40.

These tests were administered to 442 pilot cadets at Lackland Air Force Base, San Antonio, Texas. The test scores were intercorrelated, and the resulting matrix was factor-analyzed. A variation of the multiple-group method of analysis was used, with the solution being iterated twice to stabilize

both the communalities and the weights applied to the variables. The resulting factor matrix was then rotated to oblique simple structure. Eleven factors were found in this battery of 40 tests. The computation of the correlation coefficients and the extraction of factors were carried out on an IBM Card-Programmed Calculator; the rotation was accomplished by use of the Matrix Rotator at The Adjutant General's Office in Washington, D. C.

Three of the eleven factors—Rote Memory, Meaningful Memory, and Span Memory—are rather clearly defined memory factors; the nature of a fourth memory factor is not so clear-cut. The reference tests clearly identify three other factors as Verbal Comprehension, Numerical Facility, and Perceptual Speed. The eighth factor is a doublet representing the specific variance of tests 4 and 5, which are parallel tests. The remaining three factors all appear to involve visual tasks; these factors seem to represent Visualization, Spatial Relations, and Visual Memory, but the identifications are uncertain since there seems to be confounding with Deduction and Psychomotor Coordination factors.

The conclusions drawn from this study may be stated as follows.

1. Factors were found which correspond to the hypothesized factors of Rote Memory, Meaningful Memory, and Span Memory. The fact that Rote Memory is distinct from Meaningful Memory has been suggested before, by Katona (1940), McGeoch (1928), and Jones (1951), but never clearly demonstrated by factor-analytic techniques. Also, the separation of Span Memory from Rote Memory was suspected but not too convincingly demonstrated, e.g., French (1951, p. 220 and p. 246), Jones (1951).

2. All three of these factors are general for both visual and auditory presentation of material.

3. Both Rote Memory and Meaningful Memory are general for the three types of tests used—recognition, completion recall, and free recall; Span Memory is possibly restricted to free-recall tests.

4. Meaningful Memory is probably general for both verbal and nonverbal material, while Rote Memory and Span Memory are possibly restricted to verbal material.

5. The evidence with regard to the hypothesized factor of Visual Memory is equivocal; neither positive nor negative conclusions about it may be drawn.

6. The possibility of an Auditory Memory factor is suggested by this analysis.

Recommendations for Further Research

Many problems on which further research is needed have been suggested by this study; these problems seem to fall into six categories. The categories, together with some of the areas for study within each category, will be given below.

1. *Test development.* Many of the tests constructed for this study need to be improved before they are used again. The difficulty level of several of the tests needs to be adjusted; test 4 was too difficult for this population, while tests 2, 7, 11, 12, 21, and 27 were too easy. While most of the tests were satisfactorily reliable, some were not; it is likely that the factor content of the tests which had extremely low communalities in this study would be clarified if the tests were made more reliable.

2. *Clarification of factors.* To clarify the nature of the Rote Memory factor, it would be desirable to develop additional recognition and free-recall tests to measure Rote Memory; also it is necessary to determine whether or not this factor can be measured by the use of nonverbal material.

With regard to the Meaningful Memory factor, an attempt should be made to construct other recognition and free-recall tests which will measure it better than did tests of these kinds used in this study. It is possible, of course, that the test revision recommended in (1) above will itself raise the factor coefficients of the recognition and free-recall tests that were measures of Meaningful Memory in this study. Another point which needs to be checked is whether or not this factor can be measured by nonverbal material. The most critical problem, however, is to design experiments which will distinguish between different possible definitions of "meaning."

The nature of the Span Memory factor needs to be clarified by answers to the questions as to whether or not this factor can be measured only by free-recall tests, and whether or not it can be measured only by verbal material.

It might well prove helpful in investigating the nature of all of these factors to get introspective reports from subjects who are either very high or very low on one or more of these factors.

3. *Identification of new factors.* Another study is necessary to answer the question as to the existence of a Visual Memory factor; this new study should include reference tests which will clearly separate out the variance due to the previously discriminated factors of Spatial Relations and Visualization.

This study has suggested the possibility of an Auditory Memory factor; attempts should be made to clarify further the nature of this factor and to determine whether or not it is limited to verbal tests. It is possible that it might be related to one or more of Karlin's factors (1941, 1942); in any case, further investigation is needed into the nature of auditory memory factors and their relationship to other memory factors.

If it is possible to demonstrate the existence of Visual Memory and Auditory Memory factors, might it not also be possible to find one or more Kinesthetic Memory factors? This is another possibility that should be explored.

4. *Validity studies.* It is highly probable that measures of these different

memory factors will be useful in many problems of differential prediction of human behavior; in particular, they will probably improve our ability to make differential predictions of success or failure in educational and occupational situations. Studies are needed to explore these possibilities.

5. *Testing methods.* Investigation is needed in order to determine the effect of the method of testing on the factor content of the memory tests. In particular, three methods might be investigated.

The first method is one used to measure learning; traditionally learning has been measured by some variable such as amount of time or number of trials required by the subject to reach an arbitrarily defined criterion of learning. A better method, however, would be to plot an individual learning curve for each subject and to define the learning score or scores in terms of the parameters of that curve. If such parameters were used as scores, individual testing would probably be necessary.

The second testing method is one used to test forgetting; traditionally, forgetting has been measured by the amount retained by the subject at the end of a fixed interval of time after some arbitrarily defined criterion of learning has been reached. Probably the best way to establish the criterion of learning would be to plot a learning curve for each subject and to define the criterion in terms of the parameters of that curve. It would also be possible to plot an individual forgetting curve for each subject and to use the parameters of that curve as scores for forgetting. Again individual testing would probably be necessary.

In the third testing method it is the amount retained by each subject at the end of an allotted amount of study time that is measured; this is the method most often used in group testing. In this method each subject is free to allocate his study time for a test in the way which seems to him to be most advantageous rather than being forced to allocate it in accordance with a schedule which has been predetermined by the experimenter.

A comparison of the factor content of memory tests adminstered by these three testing methods should be helpful in interpreting the nature of the factors themselves as well as in relating and interpreting the results of the many studies which have used these methods.

6. *Other types of memory.* This study dealt only with relatively immediate intentional retention; work needs to be done on both incidental and long-term memory. Questions as to how many factors are involved in each of these domains and as to whether the nature of those factors will be similar to, or different from, those found in the domain of immediate intentional memory remain to be answered.

Developments Since Completion of This Study

The analysis of the data collected in this study was completed in 1954

and was reported in an Office of Naval Research Technical Report (Kelley, 1954), but due to unavoidable circumstances its publication was delayed until this time. In the intervening time there have been several further studies and reports which bear on the memory factor domain. The attention of the reader is particularly called to the reports by Allison (1950), Christal (1958), Faulds (1959), French (1963), Guilford (1957), Ross (1961a, 1961b), and Stake (1962).

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