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LEARNING PARAMETERS, APTITUDES, AND ACHIEVEMENTS

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LEARNING PARAMETERS, APTITUDES, AND ACHIEVEMENTS

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PREFACE

This monograph is a slightly expanded version of the doctoral dissertation which I wrote as an Educational Testing Service Psychometric Fellow and student at Princeton University. The members of my dissertation committee are named in the next-to-last paragraph of this preface.

The help of many people is enlisted in carrying out a project such as the present one. Among the many whose assistance has been so generously given, I wish to thank Mrs. Richard Reser, Mrs. Stanley Daugert, Dr. Lynette Saine and Dr. Paul Clifford for their participation in the collection of the data, and the school officials, cited in the appendix, who cooperated in this endeavor. Psychometrists Ann Burkhart, Gloria Turnipseed, and Charles Fortson also assisted me in a number of ways, for which I am grateful.

For secretarial and clerical assistance, I wish to thank Mrs. Jean Roberts, Miss Jeanette McArthur, Mrs. Mary Winder, Mrs. Betty Couey, Mrs. Edna Bradbury, Miss Lois Righter, Mrs. Mary Anne Exner, Miss Shirley Sutton, Mrs. Ann King, Mrs. Sally Matlack, Mrs. Kathryn Neil, Mrs. Lorraine Hubbs, and Miss Mary Kay Coonrad.

After doing some of the computation by hand, I estimated that the total analysis would have taken 192 years that way. The extension of the analysis made possible by the electronic computer is obvious. My appreciation for such computational facilities cannot be overstated. The assistance of Mrs. Ruth Blackman, Mrs. Gladys Brinckloe, Miss Henrietta Gallagher, Mr. Jack Rhubart and Mr. Ivan Strickler facilitated this work. Mr. Carl Helm was a patient tutor in teaching me computer programming techniques.

I am particularly indebted to Harold Gulliksen whose previous work laid the foundation for this study and who contributed much in the way of guidance for this project; to Ledyard Tucker who devised many of the procedures used in the analysis; to Jack Robinson who did his best to get me to make this research a worthwhile contribution to experimental psychology; and to Warren Findley who did his best to get me to make this research a worthwhile contribution to educational psychology.

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ROBERT E. STAKE

July, 1960

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CHAPTER I

INTRODUCTION

Learning is a central issue in the scientific consideration of the behavior of organisms. Few, if any, psychological theories have failed to list it as a major aspect of behavior. Indeed, many psychologists have elevated learning to a pre-eminent status. The scope of learning studies has been great. Certain problems have been studied with precision. The reports of research on learning have been numerous.

Whereas a large portion of theory and experiment on learning phenomena has dealt with general trends, the role of individual differences in this domain has been touched on only occasionally. This monograph is the report of a study of relationships among individual differences in certain learning performances and in measures of various mental abilities and achievements. By applying factor analysis and somewhat refined techniques for the measurement of learning, the author has attempted to clarify relationships between learning ability and performances on a number of conventional tests. An attempt has been made to discover the degree to which any general learning ability pervades all learning performances.

Learning Theories

The research reported here was not designed to test the specific holdings of any one of the many learning theories. To the extent that any given learning theory acknowledges individual differences, this research should aid in its assessment and improvement by illuminating some of the influences and limitations of those differences.

Most learning theories attempt to specify the necessary and sufficient conditions for learning and to offer a set of laws that can be used to explain and predict behavior without extensive reference to individual differences. As Cronbach (1957) points out, the theories have been tested almost exclusively by experimental psychologists. Typically, these experimentalists have *controlled* as many differences among subjects as possible and searched for relevant learning variables for homogeneous learners. But, the search for relevant variables can be conducted with the correlational techniques of the differential psychologists as well as with the mean difference tests of the experimentalists. All of the relevant experimental variables have been identified *only when* individual differences remain unchanged, except for error, during replication. As an illustration, suppose that the effect of a variable is being studied by administering treatments to an experimental group but not to an apparently equally homogeneous control group. Suppose that no difference in subsequent performance is found. The conclusion, of course, is that the variable is not relevant. Suppose, however, that an interaction exists; some subjects do markedly better with treatments, other subjects do worse. It should be concluded that the treatment variable is relevant. Thus, individual differences may reveal the relevancy of experimental variables that cannot readily be ascertained from analysis of group means.

Factor analysis determines the minimum number of factors or variables needed to account for the individual differences in performance. In the determination of factors for learning performances, this study has certain implications for one broad aspect of learning theories. Hull (1943) and Guthrie (1935) are one-factor theorists; that is, they have theorized that a single process accounts for all learning. Many other theorists have suggested multiple learning factors or processes. Skinner (1938), Thorndike (1932), Mowrer (1947), Hebb (1945), Harlow (1949), and Spence (1956) have explicitly ascribed learning to two different processes. Tolman has discussed a number of capacities (1932) or types of connections or relations (1949) which typify learning. If learning can be multi-dimensional as these latter theorists have stated, the dimensionality should be manifested in the learner-oriented scores whether it is or is not manifested in the experimenter-oriented classification of learning tasks. The *scores* should reflect the importance of each process for each task. If all performances are determined by a single process, the factor loadings for all performances should be on one learning factor. If, however, performances are determined by multiple processes, the performances should load on more than one learning factor. If the rote memory tasks and relational learning tasks of this study load on different learning factors, the idea that there is more than one learning process is supported.

In psychological experiments the difficulties in adequately controlling the antecedent conditions continue to prevent the substantiation or rejection of the different theories. The lack of resolution between views is partially due to definitions of learning, for experimental purposes, in terms of what can most easily be measured. To be sure, in order to be experimentally meaningful, learning must result in overt quantifiable responses. But the continuous functional relationships found in most theories are apt to be masked by using such gross measurements as total errors or total time. Too often learning curves have been used only as a visual supplement to tabular data. Curve parameters have been used only infrequently. They deserve more study because they may reveal quantitative characteristics that are not yielded by the score on any single trial or the sum or difference of selected trial scores.

Most curves are characterized by more than one parameter. Although learning has been characterized by a single parameter or measurement by many investigators, a set of two or three or more parameters may be necessary to describe a performance adequately. It is easy to imagine that the speed with which a task is learned may be independent of the rate of change in performance. For example, one learner may reach a criterion early, but his plot may show more gradual curvature than that of a learner with belated insight. And both of these characteristics may be unrelated to the regularity of the performance. Maier (1939, 1940) has suggested a variability of performance due to a need of the performer to be variable. Speed of learning, learning ability, and regularity of performance are readily defined as mathematically independent curve parameters. One major purpose of this study was to determine whether or not these potentially independent learning parameters are in fact uncorrelated with each other.

Learning Ability, Mental Functions, and Intelligence

There are a number of hypothesized mental functions, such as memory, perception, symbolization, and reasoning. Learning is one of these mental functions. The manifestations, as well as the definition, of learning, do not, however, clearly distinguish it from the others. If there is memorization, there must be learning. If there is symbolization, learning must have preceded it. But this is mere speculation, and speculation has done and will do little to right the disarray in the study of mental functions. Psychometric measurement of the functions has resulted in some order, and further exploration, enlightened observation, analysis and evaluation promise to trace out more of the complex relationships.

The ability to learn and intelligence have often been considered synonymous terms (Buckingham, 1921; Dearborn, 1921; McGeoch, 1942; Piaget, 1947). There is widespread opinion that the best way to determine the "intelligence" of an individual is to observe him in a situation that calls for learning or adaptation. The converse is generally accepted in our schools; namely, that the ability to learn, the ability to acquire information, the improvability of the child, is essentially that which is measured by an intelligence test. To be sure, there are many examples of substantial correlation between school course marks and intelligence test scores. There is little evidence, however, that, given a more rigorously controlled learning situation, the psychometrically intelligent subject will out-perform his less gifted peers. Furthermore, as Woodrow (1946) points out, since the acquisition of certain motor skills shows no appreciable correlation with intelligence test scores, we cannot simply state that improvement with practice is identical with some hypothesized intelligence. The conclusion that Woodrow favors is that there are specific learning abilities for various tasks. The possibility remains, however, that there is a general learning ability, independent of what intelligence tests measure, that is influential by itself or jointly with other factors in every learning situation.

Learning Ability and Achievement

A close relationship between learning ability and scholastic achievement has generally been assumed on the basis of their mutual relationship with measured intelligence. In the schools, the teacher is almost continuously impressed with performances that bear out the interdependence of these three. From his advantageous position the teacher observes each child's scholastic endeavor and evaluates this endeavor in course marks. It is no secret, however, that this evaluation is based on multiple standards. Sometimes the child is compared to an ideal child, sometimes with an "average" child, sometimes with his classmates, and sometimes with himself. Sometimes class marks are used to "promote the general welfare" rather than to report specific achievements. Sometimes the class marks appear to be highly influenced by incidental factors, such as intelligence test results or personality variables. Course marks are known to be erroneous indices of achievement; nevertheless, they remain the basic measurements of scholastic achievement. Although course marks are predictable to a useful extent, there is ample room for improving upon the predictive efficiency. It seems that a reasonable hypothesis might be that the prediction of future course marks could be improved by sampling the pupil's "achievement" in a controlled learning situation.

Some of the objection to course marks is lessened by the use of standardized achievement tests. These tests have the disadvantage, however, of being good tests of "intelligence," for they and the common group intelligence test measure similar general accomplishments. The relationship between achievement-test scores and laboratory learning performances is not clear. No studies of this relationship were found in the literature. In short, although the accepted indices of achievement have their shortcomings, further study of their relationships with learning ability seems justified both on theoretical and practical grounds.

The Measurement of Learning Ability

For this study, learning has been defined as the process by which there is improvement in intellectual performance upon exposure to a problem or task. This improvement must be that which cannot be reasonably attributed to some physiological or psychological events, *e.g.*, maturation or recovery from injury, which are not evoked by the learning problem or task. In this study, as in many laboratory studies on human learning, exposure will be limited to several repetitions within a single hour. The learning in this period may indeed be called "short-term" learning, and may or may not be identical with learning over a longer period.

There are three ways in which improvement in performance in human learning studies has been measured in the past. The first uses a single test. Assuming that the accomplishments of the subjects would be negligible before the learning session begins, the experimenter measures the level of performance at the end of the session. A typical memory experiment falls into this category. Since extensive study of memory has been undertaken, the findings are considered pertinent to the study of learning abilities in general. It has been found that the intercorrelations of memory tests are almost always positive but often small (Garrett, 1928; Anastasi, 1930). Further, it has been found that memory is not a unitary ability, but is probably three or more abilities, depending in part upon the various aspects of the content of the material to be memorized (French, 1951; Kelley, 1954). In those correlational studies in which learning ability has been measured with a single test on each task there is little evidence of a general learning ability.

A second method has been used in which performance has been evoked repetitively until some criterion performance has been attained. Here the experimenter feels that there were no important individual differences in pre-experiment performance and therefore measures the improvement in terms of total trials, total time to learn, or total errors. In a typical experiment, Garrison (1928) found that the correlations between course marks and scores on the Peterson rational learning problem were higher than the correlations between intelligence scores and those learning scores. In many similar studies it is found that the correlations between course marks or intelligence test scores and learning scores tend to range from around zero to about .5. Usually the so-called rational learning experiments can be expected to yield higher correlations with intelligence-test scores than the rote learning studies.

A subject's improvement during a training session is often measured in a third way by comparing the final score with the initial score. Woodrow (1938a; 1938b; 1938c) found that on a number of activities such as horizontal adding, rearranging letters to make words, or cancelling letters with complex instructions, the gain score did not, in general, have a significant relationship with intelligence as measured by the Thorndike CAVD or the Otis Advanced Examination. He reported that "both initial and final scores had exceptionally high reliability [.9 +], and the practice was long enough so that for the most part the individual learning curves showed a pronounced flattening out toward the end of the practice." Dysinger and Gregory (1941) found that the improvement during a semester of elementary psychology instruction, as measured by the gain in scores on parallel tests, correlated −.06 with scores on the Army Alpha (Nebraska Revision). On the basis of many similar studies it appears that gain scores and intelligence scores usually have positive but quite low coefficients of correlation.

A close look at the gain score reveals that it is a crude and biased measure of improvability. It is generally lacking in reliability and the correlation between it and other variables is often severely limited by the experimental design and procedure. These conclusions are easily derived from equations given by Gulliksen (1950) or Peters and Van Voorhis (1940) and are well stated by Woodrow (1946) and Lord (1956). When the limitations of this gain score as well as the limitations of the crude measures of improvement of the other two methods are fully considered, it is not surprising to find that the true relationships between laboratory learning and psychometric variables in general remain undisclosed.

Gulliksen (1934), Woodrow (1946) and others have suggested the use of learning-curve parameters to improve the measurement of learning. Woodrow fitted sigmoid learning curves to the data of two groups of subjects who had performed in the learning situations mentioned above. He calculated (a) the maximum slope of the curve, (b) the intercept of the asymptote of the curve, (c) the intercept at time zero; and he derived a fourth parameter to express absolute gain. Considering the slope parameter to be the best expression of ability to improve, he concluded that there was no general learning ability present. None of the parameters was closely related to the intelligence-test scores. There was some inter-task correlation for the different parameters, but even on parallel tasks the intercorrelation between learning rates was low (.36). It is believed that the matrix involving these parameters was not factor analyzed. The use of such an analysis and the study of learning calling for more cognition are the major differences between Woodrow's studies and that reported in this monograph.

Factor analyses have been carried out on data including learning scores, but the few new dimensions which were found were not substantially defined. Perl (1934) found one new factor and Edgerton and Valentine (1935) found two new factors in learning mirror drawing. In another factorial study, Simrall (1947) further established the case that gain scores and intelligence scores are not closely related. She used only tests of spatial and perceptual abilities and intelligence, but measured and analyzed the scores thoroughly. All her ten hypotheses pointing to close relationships among gain scores and "intelligence" were rejected. Woodrow (1939) found that the dimensions of the initial scores were no different from the dimensions of the final scores, and, recognizing that gain scores are completely dependent, concluded that "no factor consisting of an ability to improve, if by improvement is meant an increase in the goodness of scorable performance, will ever be discovered, which is distinct from the factors found in tests given just once." Such conclusions seem unwarranted. Woodrow was using tasks that involved little more than perceptual speed and number ability. A more inclusive survey of learning situations and a more incisive measuring procedure might invalidate this conclusion.

Allison (1960) has worked on a factorial study of performances of Naval recruits in learning situations, but he had not completed the study when this review was written.

It is believed that the literature does not contain a single report of a systematic study of the different relationships between the content of the material to be learned and the over-all learning performances. Furthermore, it is not clear whether or not the experimental methods used themselves have contributed to an interaction variance with learning ability. The present experiment was undertaken with an intent to contribute some basic information about individual differences in learning abilities, test performances, and academic achievements, with appropriate attention to the content and experimental presentation of learning materials.

The Plan of This Study

In order to investigate these individual differences among learners, a number of short-term learning tasks were proposed. In many ways these tasks simulated scholastic tasks. The successive scores obtained from repeated evaluation would be fitted with a mathematical curve, one curve for each learner on each task. Certain parameters of each curve then would serve as descriptive statistics for that learner on that task.

Curve parameters, conventional aptitude-test scores, and conventional achievement-test scores would be subjected to factor analysis. The factor matrix would be rotated to isolate any factors different from those obtained from conventional tests. Interpretation of the correlation and factor matrices would clarify the questionable relationships.

Specifically, the hypotheses to be tested in this study were:

1. Learning-curve parameters are significantly correlated with measured intelligence, aptitudes, and achievements.
2. Upon factor analysis, learning-curve parameters will load on some factors which are independent of factors determined by tests given just once.
3. With "intelligence" controlled, there is a partial correlation between learning ability and academic course marks that can be used to improve the prediction of scholastic success.
4. A general learning ability, orthogonal to the space defined by tests given just once, pervades performances on widely different tasks.
5. On a given task there is no significant correlation between total errors (intercept of asymptote), rate of learning (degree of curvature), and regularity of performance (mean squared deviation) when these characteristics are expressed as learning-curve parameters.
6. Learning-curve parameters from verbal tasks load on a verbal learning

factor whereas parameters from nonverbal tasks load on a nonverbal learning factor.

7. Learning-curve parameters from relational-learning tasks load on a relational-learning factor more than on a rote-learning factor whereas parameters from rote-learning tasks load on a rote-learning factor and not on a relational-learning factor.

8. Learning-curve parameters load on factors which are related to the degree to which the experimental circumstances enhance the level of motivation.

It should be noted that this research differs both in plan and purpose from that in which the changing patterns of abilities contributing to proficiency are determined by factoring scores successively at each stage of the learning (Fleishman, 1957). Here the focus is on the differences between patterns of improvement. These differences may be independent of differences in proficiency at any stage of training.

CHAPTER II

INSTRUMENTS AND PROCEDURE

The Experimental Design

There are many variables which contribute to the over-all learning situation. Four variables which are generally associated with learning performances are (a) the nature of the stimuli to which the subject is asked to attend, (b) the subset of abilities or processes that the subject is expected to use in order to perform successfully, (c) the motivation of the subject, and (d) the procedure or learning environment that that experimenter establishes. The selection of learning tasks was based upon these four variables.

Verbal stimuli were selected for half of the learning tasks; nonverbal stimuli for the other half. The nonverbal material consisted of numbers and pictures. Position cues were much more important in nonverbal than verbal tasks.

Half of the tasks were devised so that they could be solved by rote memorization. The other half called for the discovery of relationships or generalizations. In some of the latter tasks a perfect score was ultimately possible by rote memory alone, but it was assumed that superior scores would indicate the influence of rational processes.

Motivation of human subjects tends to be related to the learning environment that is provided. Therefore variables (c) and (d) above were included in the design as a single variable. Experimental conditions were changed from task to task in ways which reasonably might motivate the pupils. In the least motivating circumstance the tasks were administered in the pupils' class room just as conventional group tests are administered. The tasks were group tasks which require paper-and-pencil responses. In the second condition, tasks were similar or parallel to those in the previous condition but they were individually administered in more private quarters. The pupil set the pace, generally, and the psychometrist offered some encouragement. The hypothetically most-motivating circumstance called for game-like tasks that involve brightly colored objects, mechanical gadgets, lights and bells. The tasks were individually administered with warmth and enthusiasm. The subjects were rewarded with paper chips which later were exchanged for a small amount of money.

Figure 1 schematically represents the experimental design of the learning tasks. Its three dimensions are the experimental variables just considered.

The twelve tasks, each of which is more fully explained in the next section, are represented by the numbered circles in Figure 1. They are as follows:

Task Number	*Task Title*
1	Choice Board I, (word match)
2	Word Groups
3	Jungle Maze
4	Choice Board II, (position-number match)
5	Word Memory I, (verbs)
6	Listening I, (Tom's errand)
7	Figure-Shape Matching (circus)
8	Number Pattern I, (hexagon)
9	Word Memory II, (adjectives)
10	Listening II, (Daniel Boone)
11	Picture-Number Matching (sea life)
12	Number Pattern II, (triangle)

The Learning Tasks

The learning tasks were devised to meet the specifications of the experimental design introduced in the previous section. Since it was intended that

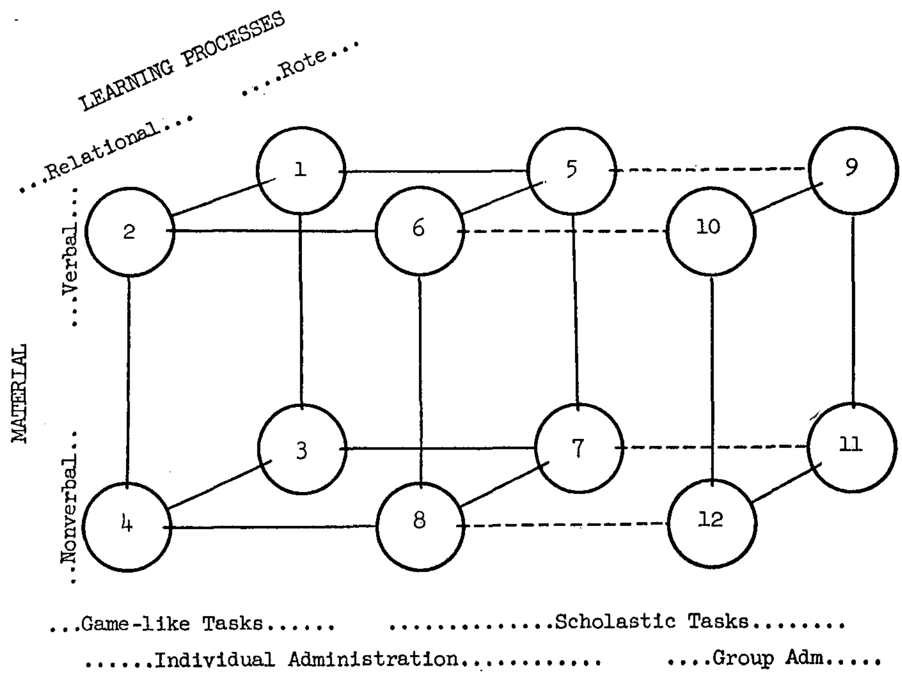


Figure 1
Three-Dimensional Schematic Diagram of Learning Tasks.

reading ability should not determine a learning score, directions for all tasks were given aloud. In general, responses were called for that would demand little in the way of ability to communicate. An x mark, a point of the finger, or a nod of the head was acceptable in many cases. Behind the formulation of each task was the assumption that every school child could completely master the task if given unlimited time to do so.

Equivalent alternative activities were used in individually administered tasks with successive children so that there would be less likelihood of effective coaching. The children were not asked to keep the activities a secret, because such a request might have caused more dissemination of vital information than otherwise. Also to minimize the effect of inter-child coaching, "slow" and "quiet" children were tested first, and the psychometrist avoided any recapitulation of the proceedings with the child. The latter's attention was usually easily directed to the accumulation of reward chips at the end of Tasks 1–4.

The twelve learning tasks are described below, with particular emphasis on those characteristics which seem most likely to determine the task's place in the multi-dimensional space of a factor analysis. None of the tasks had been used as a learning task in previous studies.

Task 1, Choice Board I. This task is a rote memory test for individual administration. On the face of the board are nine positions, each of which has a push-button switch and a small red light. For this task the simple words shown in Figure 2 are placed as shown, one at each position. The

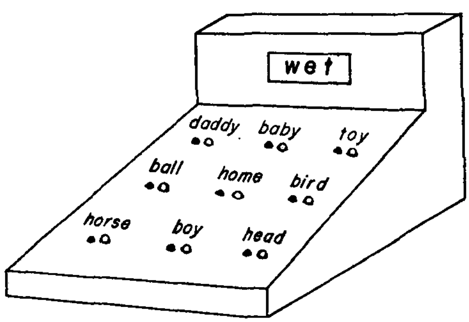


FIGURE 2
Choice Board I.

child is to press the switch by the word which, by rote assignment, matches the stimulus word in the window. If the response is correct, the red light at that position goes on, a chime sounds, and a paper chip is awarded. If the response is incorrect, the red light goes on at the position of the *correct* response. Thus, corrective information is available following each response.

There are nine different window stimulus words, one paired with each word on the board. The pairing is the same for all children. A trial consists of nine responses, one for a presentation of each window stimulus. The stimuli are presented in the same order on all trials. All subjects complete six trials, regardless of their success.

Task 2, Word Groups. In this task the subject is directed to examine the four words on a card and to choose one of four boxes in which each card should be placed. The psychometrist indicates the correct choice if the subject's choice is incorrect. Unknown to the subject the boxes are assigned to the following classifications: white things, household furnishings, common

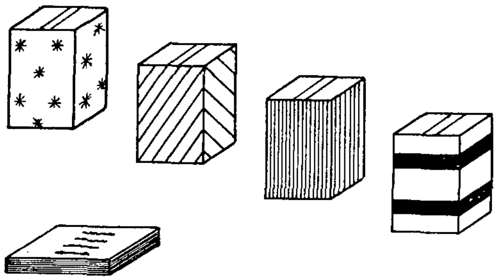


FIGURE 3
Materials for Task 2, Word Groups.

edibles, and living things. Any or all of the words on a card might be ambiguous alone, but as a group of four only one classification is correct. Sample cards based on flowers and colors are used as part of the instructions.

A trial consists of five responses and each subject completes nine trials. Because the first few responses are determined by chance the first trial is not counted. A paper chip is awarded for each correct response. The colors of the boxes assigned to the classifications are changed occasionally.

Task 3, Jungle Maze. This task is a rote memory, nonverbal task for individual administration. The maze is an electric board with remote controls. The child is acquainted with the location of the starting point and goal and with the fact that at each of the seven choice points there are two button switches. When a button is pressed, a pathway, sometimes circuitous but never ambiguous, is illuminated and the subject must follow that pathway to the next choice point. The task is completed when the subject has traversed the maze six times, regardless of the number of responses. Only

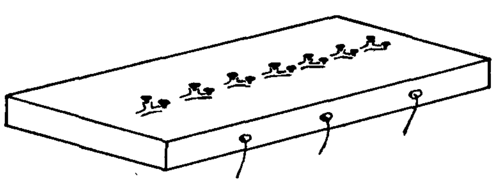


FIGURE 4
Jungle Maze.

the first 150 responses are analyzed when that number is exceeded. Performance is measured in terms of progress made on successive blocks of five responses each.

The apparatus is wired so that the initial response at each point is an incorrect response. At the time this *initial* response is made the psychometrist operates the remote control to make the unselected pathway correct. Such an arrangement was felt to be necessary to neutralize the effects of coaching. It has the added advantage of insuring that every pathway is explored on the first traverse. The two advantages are somewhat offset by the fact that continued position habit forces slightly poorer-than-chance performance and the fact that some solution sequences are easier to remember than others, (*e.g.*, RRRRL instead of RLLRL).

The majority of pathways do not lead to the adjacent goalward choice point. Many, in fact, end at the same or a previous choice point. It is usually not immediately apparent whether or not the response has been a correct one. Paper chips are awarded upon reaching the goal.

Task 4, Choice Board II. This task utilizes the same board as Task 1 except that no words appear on the face of the board. There are from one to

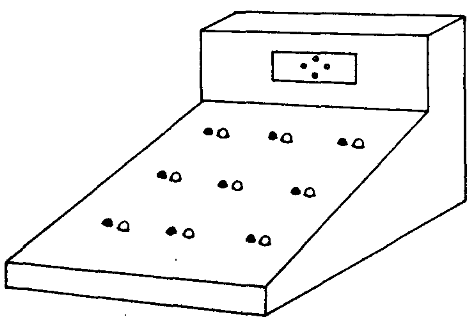


FIGURE 5
Choice Board II.

nine black dots on the window stimulus cards. The subject is told that only one button is correct for each stimulus in the window. When the correct button is pressed the adjacent light lights, the chime chimes, and the chips are awarded. When an incorrect response occurs the light at the correct position goes on.

The association of stimuli to positions is established by determining a simple order of positions like:

7	8	9		9	2	3
6	5	4	or	8	1	4
1	2	3		7	6	5
Pattern S				Pattern I,		

and pairing of stimulus of n dots with position number n. Thus, with Pattern S when three dots were displayed in the window the correct button would be the button on the lower right. Different patterns are used for different children.

Two different arrangements of each number of dots provides the experimenter with a deck of 18 stimulus cards. Two positions and the corresponding four cards are held back for testing to distinguish between a rote and an inductive solution. Each trial then consists of 14 responses. The task is terminated after six trials.

Task 5, Word Memory I. This is a paper-and-pencil task for individual administration, requiring the subject to listen to the 16 simple verbs in Figure 6 in order to write down as many of them as possible, in any order. This procedure is repeated six times with the same words read in the same order

burn	run
eat	came
bite	fly
begin	love
add	miss
blow	call
break	buy
try	live

FIGURE 6
Words Used in Task 5, Word Memory I.

each time, at about one word per second. The subject does not have access to his work on previous trials nor to any evaluation of his success.

Generous standards are used for scoring the responses, particularly when the subject has a general inability to spell.

Task 6, Listening Comprehension I. This verbal, individually administered task utilizes one selection of the Cooperative STEP Listing Test, Level 4. The selection, a story about a child's errand, is read to the subject. The same ten multiple-choice questions, adapted from the STEP questions, are presented to the child after each of six readings. The questions themselves are not read by the psychometrist because it was desired that the task be parallel, except for external conditions, to Task 10.

Reading time for the selection is about one minute. Subjects are allowed, within reason, as much response time as they desire. They are directed to indicate their response by encircling one of the four choices following each item stem. The majority of items were devised to test comprehension of implications rather than awareness of explicit statements.

In Task 6 the child receives no direct information as to the results.

Task 7, Figure Shape Matching. This task is a nonverbal, rote memory task for individual administration. For thirty seconds the subject is exposed to a display of twelve circus silhouettes, *e.g.*, lion, juggler; each of which is bordered by a recognizable shape, *e.g.*, heart, circle. Then the subject faces a display of the six unique shapes which are used; and, taking unbordered silhouettes mounted on clear plastic one by one, he attempts to match each of them with the assigned shape. He is given no information about the appropriateness of his pairings, other than what he gets from the subsequent training phase.

The subject may state his pairing orally, may point to the shape that he wishes to pair with the silhouette under consideration, may hold the silhouette up to the shape so the visual image is similar to that of the training display, or otherwise indicate his choice in any way that conveys his meaning to the psychometrist. The child is not permitted to examine other silhouettes than the one provided him at that time.

In Task 7 there are six trials of twelve responses each.

Task 8, Number Pattern I. This task is a nonverbal task which is similar to many rote-memory tasks. The stimuli for this task are single-digit numbers which have been arrayed as in Figure 7. Exposure to the stimuli lasts thirty seconds. Several minutes are permitted thereafter in which the child is directed to write the stimulus numbers into a similar layout of blank cells. This procedure is repeated six times, yielding six trials of 37 responses each.

Unlike the usual rote memory experiment, however, the stimuli here have been arranged in a pattern that follows simple, if not always obvious, rules. If the subject were told a rule or two and told where to begin, it might be expected that he would make all responses correctly without a glimpse at the stimuli. Many insight tasks seem to be solved not only by methodically testing hypotheses but by being lucky in dreaming up the right hypothesis; here, however, there may be many partial insights that permit a good showing.

The score is based on number of cells properly marked. This does mean that proper placement is necessary before pattern recognition can be acknowledged. A child who reproduces the pattern of numbers accurately but places all entries one cell to the right, or left, gets a very poor score. To emphasize relationships, it would be better to evaluate the pattern itself but this necessitates undesirable subjective judgments by the scorer.

Here, as in all the tasks in the battery, there is no determined effort to

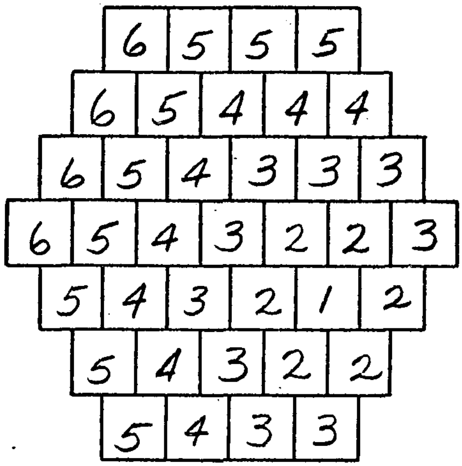


FIGURE 7
Arrays Used in Task 8, Number Pattern I.

get the child to fill every blank. Scores are total rights, with no correction for guessing.

Task 9, Word Memory II. This task is parallel to Task 5, except that Task 9 is for group administration. The sixteen simple adjectives used in this task are listed in Figure 8.

Task 10, Listening Comprehension II. This task is parallel to Task 6 except that Task 10 is for group administration. The story used in this task is the Cooperative STEP 4 Listening selection concerning Daniel Boone.

angry	six
dry	green
big	funny
fat	many
busy	first
cold	good
dear	high
clean	holy

FIGURE 8
Words Used in Task 9, Word Memory II.

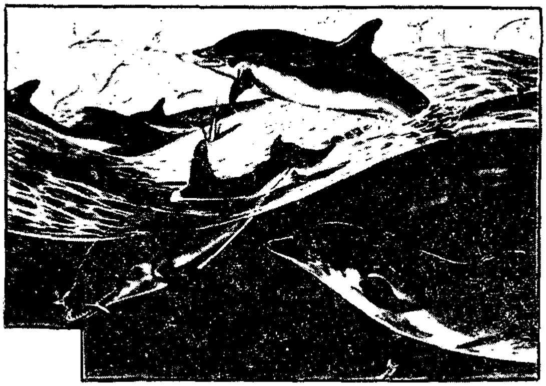


FIGURE 9
Sample Picture Used in Task 11, Picture-Number Matching.

Questions and answers were not read to the pupils because, in pretesting, certain response choices evoked laughter or head nodding that undoubtedly influenced their desirability.

Task 11, Picture Number Matching. This nonverbal, rote memory, group task is a modification of a picture number test used by Anastasi. In Task 11 a two-digit number has been randomly assigned to each of a set of pictures of sea life, such as the example in Figure 9. A picture is projected on a screen before the class for eight seconds. It is immediately followed by the next one, then the next one, and so on until all 16 pictures have been seen. Immediately thereupon, the child opens his test booklet to a page of 15 of

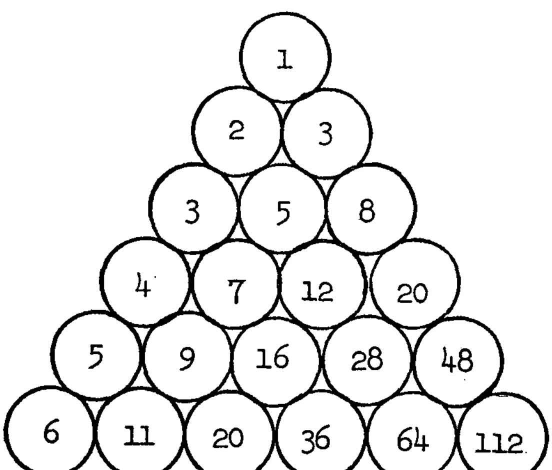


FIGURE 10
Arrays Used in Task 12, Number Pattern II.

the pictures, with the attached boxes blank. He is given several minutes to fill in the proper numbers.

This routine is repeated six times, with the order of the projections and the arrangement of the pictures on the test page remaining unchanged from trial to trial. At the end of a trial the completed page is collected. The child is given no indication of his success other than what he can ascertain by watching the numbered pictures the next time around. The children are given an opportunity to go through the training and testing procedure (using a picture of a monkey) before the first training phase is commenced.

Task 12, Number Pattern II. This task is parallel to Task 8 except that Task 12 is for group administration. The pattern of numbers used in the group situation is shown in Figure 10.

The Reference Battery

The following mental abilities were considered to be the most likely factorial correlates of learning performances and were therefore subjected to measurement by reference tests:

1. verbal ability,
2. numerical ability,
3. rote memory,
4. inductive reasoning,
5. arithmetic reasoning,
6. spatial ability,
7. perceptual speed,
8. attention,
9. persistence.

The factorial reference tests were supplemented by the Otis and the Kuhlmann-Anderson intelligence tests. It became necessary to omit the latter test, however, because scores on it were obtained from less than 75% of the subjects. Both the Otis raw score and the IQ were used in the analysis.

Several of the nine abilities were measured economically by the SRA Primary Mental Abilities battery for ages 7–11. Included therein are subtests of:

1. verbal meaning (reading),
2. verbal meaning (non-reading),
3. space,
4. reasoning (word grouping),
5. reasoning (figure grouping),
6. perceptual speed,
7. number computation.

The remainder of the reference battery was made up of revisions of tests previously discussed in the literature and a few original tests. They are as follows:

Number: Variable 56. A highly speeded (1 min.) test of the addition of two single-digit numbers. Reading ability is not required.

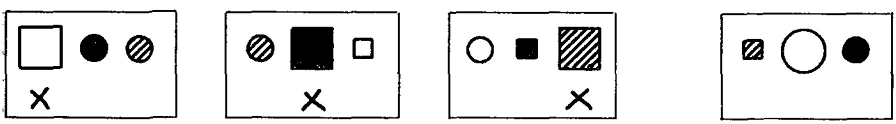
Rote Memory: Variable 49. An adaptation of Anastasi's Picture Number Test calling for a two-minute exposure to twelve rote numbered pictures and an immediate testing thereafter. The pupil responds by writing in the proper number beside each picture on the test page. A preliminary run precedes the actual testing. Reading ability is not required.

Variable 50. An adaptation of Thurstone's First Names Test. The memory page, listing twelve common full names, *e.g.*, Ann Wilson, is studied for two minutes. On the test page are the twelve last names, rearranged, with blanks preceding each for insertion of the associated first name. A four-name instructional warm-up test is used. Reading ability is required only for the names.

Variable 47. The first trial score of Task 5. Writing ability is required.

Variable 48. The first trial score of Task 11. Reading ability is not required.

Inductive Reasoning: Variable 51. An original figure-classification test with items like the one shown in Figure 11. One *X* is to be placed in the



The marked things are big squares. In the first three boxes all of the big things are marked. In the last box the X should be under the big thing, the big white circle.

FIGURE 11
Figure Classification Sample Item.

last box in accordance with the marked elements of the first three boxes, regarding size, shape, etc. Lengthy instructions are given orally.

Arithmetic Reasoning: Variable 54. Sixteen SCAT Form 5A, Part IV items, selected for a fifteen-minute test. Reading ability is required.

Spatial Ability: Variable 61. An abbreviated version of Thurstone's Hands Test with instructions adapted for oral presentation to children.

Perceptual Speed: Variable 58. Number cancelling, adapted from Thurstone. For three minutes the pupil strikes out digits identical to the first digit in that row. Directions on the test are read aloud.

Variable 59. Letter cancelling, adapted from Thurstone. For four minutes the pupil crosses out, in each group of 18 words, the three words containing the letter "*a*." Directions on the test are read aloud.

Variable 60. Number comparison. The child distinguishes between those pairs of numbers that are identical and those that are not. Directions on the test are read aloud.

Attention: Variable 64. An eight-item original test of ability to follow written directions. This test was made very easy because it was the first test to be administered for this study.

Variable 73. (Omitted in analysis.) A six-item test of ability to follow oral directions.

Persistence: Variable 74. Purportedly a reading-comprehension test, with a reading selection which becomes increasingly difficult and finally unintelligible. When they are ready to obtain the question sheet, the pupils raise their hands. The teacher secretly records study time. (Omitted in analysis because of group interactions; *e.g.*, in at least one class all sat waiting for someone else to be first.)

Variable 75. Number test, variable 56, readministered six times with problem order rearranged; similar to Pauli Test (Pauli, 1921). This was intended to parallel the learning tasks with material on which extremely little learning could be expected. (Omitted in analysis because improvement did occur over early trials. The resulting curve did not conform to fitted theoretical curve family without special transformation.)

Achievement was assessed in two ways. The Stanford Achievement Test, Complete Battery, was administered, and course marks were recorded in five subjects for the six-week period during which the reference tests were administered. The standardized achievement battery included scores on reading comprehension, vocabulary, spelling, language, arithmetic reasoning, computation, science, social studies, and study skills. A median of these subtest scores was also included in the reference battery. Course marks were available for all pupils in reading, arithmetic, spelling, language, and science.

The Sample

The sample of subjects chosen for this research was a group of seventh-grade public school children in Atlanta, Georgia. Four Negro schools and four white schools were selected so that the sample, with respect to previous intelligence-test scores, was representative of the schools in that city. One class from each school was tested. (In schools having more than one seventh-grade class, the classes were not made up by homogeneous grouping.)

There were 289 children on the rosters of these classes during the two-month span of testing. After the testing had been completed and certain tests eliminated from the analysis, it was found that complete and usable scores were available for 240 pupils, 122 Negro and 118 white. Scores on these 240 were submitted to analysis.

Administration

All original or revised tests and all learning tasks were pretested both in New Jersey and Atlanta schools. Four qualified psychometrists, two white and two Negro, were engaged to conduct all of the actual testing. To acquaint themselves with the instruments for this particular study they conducted the tests and tasks in schools not in the sample.

The schedule of tests appears as Appendix B. Usually, a session was completed in all schools before another began. All subjects took the tests in the same order, although the interval between sessions varied from one day to six weeks.

Quarters for individual testing varied from school to school, but because of school crowding, they were uniformly undesirable. Usually, the two psychometrists were testing in the same room, separated by a portable blackboard or bulletin board.

The classes received the initial trials of the group instruments as they do any group tests, but in later trials they were restive and even, on occasion, somewhat belligerent during the repetitious training phases. Almost without exception, however, they immersed themselves in the job during the response phases. They were reasonably tolerant, even sympathetic, toward the psychometrists, but anxious to finish the portion of these tests in which they were engaged. During the individual administration of the parallel tasks, their reactions were similar, and frequently verbalized.

During the subsequent administration of the game-like tasks, however, almost every child was attentive throughout all trials. There was no doubt in the minds of the psychometrists that the children found these tasks interesting and challenging.

The measures taken to preserve the security of the tests were not adequate. In spite of the initial assurances that this testing was not for the record there was some classroom cheating. On the game-like tasks far too much information passed from tested to untested pupils. For this reason some of the scores that were analyzed are unquestionably invalid. Some pupils boasted of their preparedness to the psychometrist. They were tested and dropped from the sample. The psychometrists and author concluded that such tasks as Task 2, with a few key classifications, are not adaptable for individual administration to such a group.

A grand total of fifteen to forty cents was awarded each subject. The amount depended on the number of right responses and the generosity of the psychometrist. Subjects were informed at the outset that both skill and luck would decide the amount.

CHAPTER III

PARAMETERS OF THE THURSTONE LEARNING CURVE

The Thurstone Rational Hyperbola

Quite a few mathematical functions can be used to represent learning. Ebbinghaus (1885) used a logarithmic curve, Woodrow (1940) and Culler and Girden (1951) a sigmoid curve, and Schükarew (1907) and Hull (1943) an exponential curve.* These functions were chosen mainly because they would fit the data and, thus, are considered empirical rather than rational curves. Hilgard (1956) and Hovland (1951 p. 677) have contended that rational curves, derived from assumptions about the learning process, are somewhat preferable to empirical equations. The author of this monograph agrees and, although empirical curves might fit as well, he has chosen a rational learning curve for the present consideration of the use of curve parameters to describe the progress of individual learners.

One of the best-known rational curves is that of Thurstone (1930), which was later generalized by Gulliksen (1934) and expressed as:

$$\tilde{u} = \frac{g}{c}\left[1 - \left(\frac{\frac{h}{k}}{\tilde{w} + \frac{h}{k}}\right)^{c/k}\right] \tag{1}$$

where

$\tilde{u}$ represents the cumulative errors,
$\tilde{w}$ represents the cumulative successes,
g represents the initial strength of the incorrect response,
h represents the initial strength of the correct response,
k represents the effect of reward, and
c represents the effect of punishment.

Equation 1 represents a three-parameter family of right hyperbolas. The variables are measured cumulatively. Basic to this equation is the assumption that the strength of a given habit increases by a constant increment each time the corresponding response is made. The reward-punishment-ratio

*See Lewis (1960, pp. 455–543) for a summary of such efforts.

parameter, c/k in equation 1, makes it somewhat unwieldy. Unwieldiness tends to be a characteristic common to rational curves. The advantages of such a family of hyperbolas are noteworthy, however, as answers to the following questions reveal.

There are several questions that may be considered in comparing theoretical learning curves, rational or empirical. Can it be assumed that a subject will eventually perform in an invariant way, perhaps perfectly? If the answer is yes, the curve should be asymptotic. With proficiency (or lack of it) plotted against time, as in the conventional plot, or in a cumulative plot, with accumulated wrong responses plotted on accumulated right responses, the curve of the complete learner should approach a horizontal asymptote. If the answer is no, eventual or terminal performance cannot be treated as a parameter.

Can a subject make some correct responses on the selected tasks even before there is an opportunity to learn? If yes, there is then some "chance performance" which is represented by a horizontal line above the abscissa in a conventional plot, or in the cumulative plot by a straight line with a slope not equal to zero or infinity. If the answer is no, the retrojected initial performance line in the conventional plot probably should coincide with the abscissa. For the cumulative plot this performance line would be vertical. (Correction of the response scores for chance success yields scores which are appropriate for a model based upon a situation in which there can be no chance successes. If such a correction is used, the question posed in this paragraph is not important in the selection of a theoretical learning curve.)

The hyperbola and the sigmoid, for the cumulative and conventional plot respectively, can be set to approach asymptotes which are reasonable in terms of the above questions. The logarithmic and exponential functions approach a horizontal terminal asymptote but do not originate with reference to a chance performance asymptote. Whereas these latter two functions may fit any set of learning data well, they do not have the desirable characteristic of representing the learning process as one which is limited by a chance performance on one hand and by a perfect or otherwise rigid performance on the other hand.

Is it possible for a performance to be completely random on all trials up to a certain moment and errorless thereafter? If so, the theoretical family of curves should include a member which is isometric with intersecting straight lines. The hyperbolic and sigmoid families include this isometric case, the perfectly insightful performance, as a limiting case whereas the logarithmic and exponential families do not. It should be noted, however, that the simple hyperbola or sigmoid takes care of complete insight but not partial insight. If before the radical change the performance is nonchance or if after the change the performance is not errorless, a more complex family of curves is needed. (When a task is composed of heterogeneous components, partial insights become much more likely than complete insight, and in general, the likelihood of a simple curve being an appropriate model decreases. Among others, Thurstone (1930) and Rashevsky (1948) have discussed models for more complex situations.)

The Thurstone hyperbola has, besides the advantages of a rational equation, the advantages of reasonable limiting cases both in terminal circumstances as well as with regard to insightful solutions. Its main limitation perhaps is that, as a function for cumulated data, it requires the results of all responses. If some overt or covert responses are made but not recorded, the accumulation of data can continue only on an arbitrary basis. With other curves where proficiency is a function of time (or trials), missing data are not crucial if the interval of the missing data is known.

Modification of the Thurstone Equation

For this study it was assumed that the effect of a wrong response would be equivalent in amount to the effect of a correct response, *i.e.*

$$c = k. \tag{2}$$

Equation 1 then simplifies to

$$\tilde{u} = \frac{g}{c}\left[1 - \frac{\frac{h}{c}}{\tilde{w} + \frac{h}{c}}\right] \tag{3}$$

which is the two-parameter function Thurstone originally presented. The symbols $\tilde{u}$ and $\tilde{w}$ represent true scores. When an observed score is influenced by some systematic error such as that in multiple-choice situations, a correction should be applied. This correction is equivalent to a rotation of axes to set the asymptotes at the desired slopes. Consideration of the notion of error, as taken from measurement theory, is the major departure of the formulations in this monograph from the formulations of Thurstone and Gulliksen.

The modified relationship between incorrect and correct responding is derived in the following manner. To make the correction for guessing, it is common to assume that the chance probability of a correct response, p, is the same for all stimuli or items. As usual, q, the chance probability of a wrong response, equals $1 - p$. The observed sum of right responses, w, is the true sum, $\tilde{w}$, augmented by those guessed responses which were scored as successful, which on the average will be $p\tilde{u}$.

$$w = \tilde{w} + p\tilde{u}. \tag{4}$$

The observed errors, u, are fewer than the true errors by the same difference.

$$u = \tilde{u} - p\tilde{u} = q\tilde{u}. \tag{5}$$

Substituting (4) and (5) into (3),

$$\frac{u}{q} = \frac{g}{c}\left[1 - \frac{\dfrac{h}{c}}{w - \dfrac{p}{q}u + \dfrac{h}{c}}\right]. \tag{6}$$

Solving for w,

$$w = \frac{p}{q}u + \frac{h}{c}\left[\frac{u}{\dfrac{g}{c}q - u}\right]. \tag{7}$$

If, to give the learning curve its customary negative acceleration, u is plotted on w, the curve is limited by a horizontal asymptote which intercepts the ordinate at A. When $u = (g/c)q$, $w = \infty$; therefore

$$A = \frac{g}{c}q. \tag{8}$$

Substituting (8) into (7), and letting $H = h/c$, the modified Thurstone equation is obtained.

$$w = \frac{p}{q}u + H\left(\frac{u}{A - u}\right). \tag{9}$$

Parameters

Equation (9) is a hyperbolic curve which defines success in terms of its "cost" in total errors. The parameter, A, represents the "cost" of complete learning in terms of errors made after the observations begin, including projected errors to be made after observations cease. The coefficient, p/q, identifies the slope of the asymptote emerging from the third quadrant or, in other words, the slope of the chance performance line. With u plotted on w, the slope of the chance performance line is q/p. Where 2α is the acute angle between asymptotes,

$$\frac{p}{q} = \cot 2\alpha. \tag{10}$$

The remaining independent parameter can be used to differentiate—hypothetically, at least—between two learners who approach the same asymptote at different rates. One learner might start poorly and make rapid improvement whereas a second learner starts well but improves slowly. Regardless of the number of errors each makes, the former would commit most of his errors early; the latter would distribute his errors more evenly over the period of learning. One is likely to say that the first learner has greater insight.

The first learner's curve would have a higher degree of curvature. To obtain a parameter which reveals the degree of curvature, K, it is proper to use

$$K = \frac{f''(u)}{\{1 + [f'(u)]^2\}^{\frac{3}{2}}} \tag{11}$$

where $f'(u)$ and $f''(u)$ are the first and second derivatives of w with respect to u. Specifically, by differentiating (9),

$$f'(u) = \cot 2\alpha + HA(A - u)^{-2} \tag{12}$$

and

$$f''(u) = 2HA(A - u)^{-3}. \tag{13}$$

Since the curvature changes along the curve, one can select the maximum curvature as the curvature parameter. (The maximum curvature referred to here is the maximum curvature for the total, projected curve. The maximum curvature for the segment of the curve bounded by the first and last observation would be different in many cases.) At the point of maximum curvature

$$f'(u) = \cot 2\alpha, \quad \text{and} \tag{14}$$

$$f''(u) = 2(HA \sin^3 2\alpha)^{-\frac{1}{2}}. \tag{15}$$

The maximum curvature then is found to be

$$K_m = (2HA \cot^3 \alpha)^{-\frac{1}{2}}. \tag{16}$$

To refer again to the term "insight," K_m is seen to be an index of insight. Another parameter of possible importance is the slope of the curve at the beginning of observations. This parameter is the ratio of wrong to right responses at that time. This initial slope parameter, M_0, can be obtained by setting u equal to zero in (12). Then,

$$f'(u) = \cot 2\alpha + \frac{H}{A}. \tag{17}$$

When u is plotted on w

$$M_0 = \left(\cot 2\alpha + \frac{H}{A}\right)^{-1}. \tag{18}$$

The final observed slope is another possibly useful parameter. It is designated M_t.

$$M_t = [\cot 2\alpha + HA(A - u_t)^{-2}]^{-1} \tag{19}$$

where (u_t, w_t) are the coordinates of the final point of the "observed portion" of the theoretical curve. This theoretical u_t corresponds to the observed U, the total observed errors. Both u_t and U are parameters. Of course, U already is a very commonly used parameter in learning experiments.

The coordinates of the intersection of asymptotes can be found. The u value is A. The w coordinate, B, is given by

$$B = A \cot 2\alpha - H. \tag{20}$$

The semi-transverse axis, the minimum distance from the intersection of asymptotes to the curve, is given by

$$J = (2HA \tan \alpha)^{\frac{1}{2}} = \frac{1}{K_m} \tan^2 \alpha. \tag{21}$$

Since J and K_m are inversely proportional, J would be directly proportional to the minimum radius of curvature. J can be considered a measure of resistance to change.

Thus, in the Thurstone learning curve there are several potentially useful parameters, although only two curve parameters, any two, are independent in the mathematical sense. The p/q parameter, related to chance performances, is set by the experimenter. The remaining parameters are:

- A, the total errors for perfect learning;
- U, the total observed errors;
- H, the initial strength of right response, expressed in reward-equivalent units;
- K_m, the maximum curvature of the projected curve;
- M_0, the initial ratio of wrong to right responses;
- M_t, the final ratio of wrong to right responses;
- B, A, the coordinates of intersection of asymptotes; and
- J, the semi-transverse axis, the resistance to change.

The question arises as to which two independent parameters are the most useful measures of the learning. The J parameter is essentially the same as a measure of learning obtained by the "common-points-of-mastery" method advocated by Thorndike (1928) and Ruch (1936). Wiley and Wiley (1937) advised the use of an index of learning ability which is essentially the same as J. The K_m parameter is a more direct measure of curvature, but either J or K_m can be called a curvature or insight parameter. J and K_m are the only two of the proposed parameters which are independent of the beginning or end of experimental observations.

It is more common to think of learning success in terms of total errors, U or A. Certainly a description of the performance would be incomplete without some indication of the number of errors committed. The rate of erring at outset, M_0, and at the termination of testing, M_t, provide additional information of importance.

The choice of parameters might very well be different in different learning situations. For clear differentiation between learners, it is desirable that the two parameters be not too highly correlated. Parameters that are highly correlated in one situation might be relatively uncorrelated in another.

For this study parameters A and J were chosen. They are identified in the remainder of the monograph respectively, as the asymptote and curvature parameters. The theoretical relationship between right and wrong responses with these parameters is

$$w = \frac{p}{q}u\left[1 + \frac{J^2}{2A(A-u)}\right]. \tag{22}$$

Error of Fit

Since each response, right or wrong, results in a unitary change on the cumulative learning plot, the $w + u$ dimension is errorless and the $w - u$ dimension, orthogonal to $w + u$, contains all the error of measurement. The line from an observed score to the corresponding predicted score will parallel the $w - u$ diagonal. The length of this line is the error of fit which is

$$e_i = \frac{1}{c\sqrt{2}}\left[cA + H + T_i - 2cu_i - \sqrt{(cA + H + T_i)^2 - 4cAT_i}\right]. \tag{23}$$

where the unfamiliar symbols are defined as follows:

e_i represents the error of fit for any point i,
c represents a constant and is the reciprocal of q,
u_i represents the cumulative observed errors at point i,
T_i represents the cumulative observed responses at point i.

If it can be assumed that the theoretical equation is an accurate representation of the learning, the errors of fit can be used to describe the discrepancy between learning and performance for the individual learners. But whether or not this can be assumed, the size of these errors can be used to distinguish between those who make uniform progress and those who fluctuate between much and little progress from trial to trial. For the same reasons that the standard deviation is a desirable statistic of variability, the standard error of fit, SE_i, is considered here as a regularity parameter:

$$SE_i = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2}\,. \tag{24}$$

The standard error of fit is not a parameter of the learning curve but rather a parameter of the performance. Just as the curve parameters, however, it has potential use in the differentiation between learners. In this study it was used as a third learning parameter and is hereafter referred to as the fit parameter.

Fitting the Obtained Data

There is no direct solution for the parameters, A and J, of the curve which has the "least-squares" fit. With observed errors plotted against observed right responses, first approximations of the values of A and J are obtained algebraically by passing the hyperbola through the origin, a centroid of several points in the middle of the plot and a centroid of several points at the end of the plot. In this study, three points were used for each centroid. The fit of this approximated theoretical curve to the plotted points satisfied visual scrutiny in most cases. Performances wholly along either asymptote, from which maximum curvature must be extrapolated, are often poorly estimated by this procedure, but there were almost no such performances.

Certain arbitrary decisions were made to facilitate mass analysis on data-processing equipment. If the slope defined by the origin and the final centroid was greater than the chance slope or if the curve defined by the origin and centroids did not exhibit negative acceleration, a straight line was passed through the origin and a centroid of all other points. The parameters of straight lines that would have been infinite otherwise were arbitrarily set just higher than the highest parameters that could be obtained from observed data.

It was originally intended that the curves would be fit by iterating to a least-squares solution. Iteration by Jacobian was attempted but convergence was prohibitively slow.* A double maximum slope procedure, utilizing first derivatives only, was devised by Ledyard Tucker, but this procedure also was too slow in converging.

The hope for a least-squares fit was abandoned† and the first approximations were accepted as the A and J parameter values. The standard error of fit was determined for each of the 2,880 curves by the use of equation (23). Thus, three mathematically independent parameters were obtained for each curve.

*An iteration took about 5 sec. on the IBM 650. The criterion of convergence was not realized, for most curves tried, on eight iterations.

† The computer programming for the iteration procedure was later reworked and reasonably economical solutions were obtained. By this time, however, this study had been completed.

CHAPTER IV

ANALYSIS

Correlations by Race

Because the intellectual performances of Negro and white subjects frequently have been found to differ, separate correlations by race were determined. In Figures 12, 13, and 14 are scatterplots of the 2,520 pairs of correlation coefficients. If a best-fitting line passes through the origin of such a plot it is safe to conclude that the same factor loadings could be expected from an analysis of either set of scores. Such a conclusion seemed warranted in this case.

About 16 similar scatterplots based on all correlations involving a single variable were examined. Again the assumption that the same factors were operating congruently seemed warranted. With Task 2 parameters A and J, a quite different scatter was found. Although none of the correlations was significant, the slope of the bivariate distribution here was negative. This task had already been found wanting because of its high floor, high chance performance, brevity, and vulnerability to cheating. Because of this and the peculiar plot, the results of Task 2 were considered highly questionable. Correlations of the parameters of this task were not used in the groups of variables selected for factoring by the group method.

Since it was found that the same factor loadings could be anticipated from both Negro and white children, the data were pooled to obtain a more stable matrix of intercorrelations. This matrix is presented in Appendix C.

When it was decided that a separate analysis of Negro and white correlations was unnecessary, the discrepancy in means of scores for these two groups was partialled out by factoring. An arbitrary score of 0 was assigned to Negro pupils, 1 to white pupils. This discrimination is labelled Variable 63 in the matrices in the appendices. By using Variable 63 to determine the first factor, a residual matrix was obtained which partials out the effect of race on the test scores. The same matrix could have been obtained by standardizing scores across all subjects, then calculating covariances from group means, and determining the corresponding correlations. This residual matrix is presented in Appendix C.

Factor Analysis

The 72×72 matrix of intercorrelations was factored by the multiple group method (Thurstone, 1947). Ten factors were extracted on groups

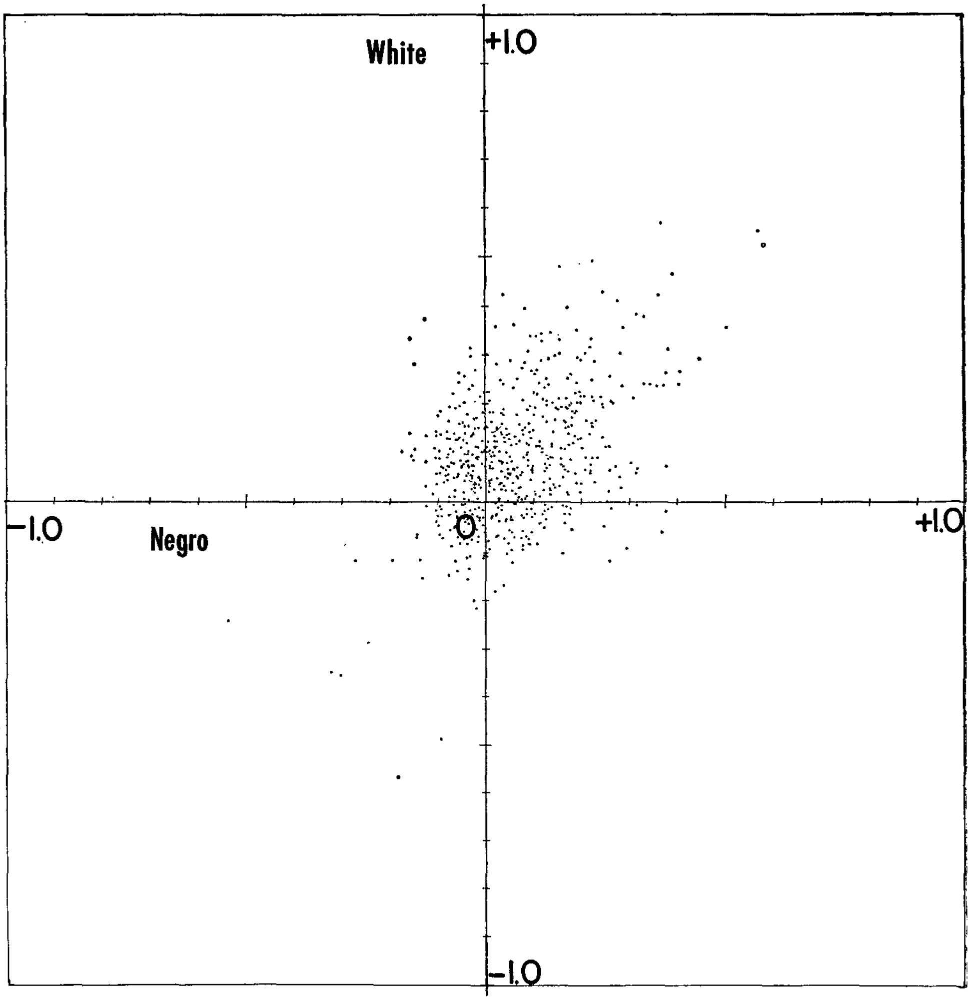


FIGURE 12
Scatter Plot of Coefficients of Correlation Between Learning Parameters for Negro Pupils (X Axis) and White Pupils (Y Axis). (Each point represents the correlation between a given pair of learning variables based on a sample of Negro pupils and a sample of white pupils.)

of reference battery variables. Examination of the eleventh matrix of residual correlations revealed that the covariance of the reference tests had been almost completely accounted for by the ten factors. Many correlation coefficients between experimental task variables still exceeded $+.1$ but only a few small groups could be found. Two groups of asymptote parameters and two groups of fit parameters were extracted. Inspection

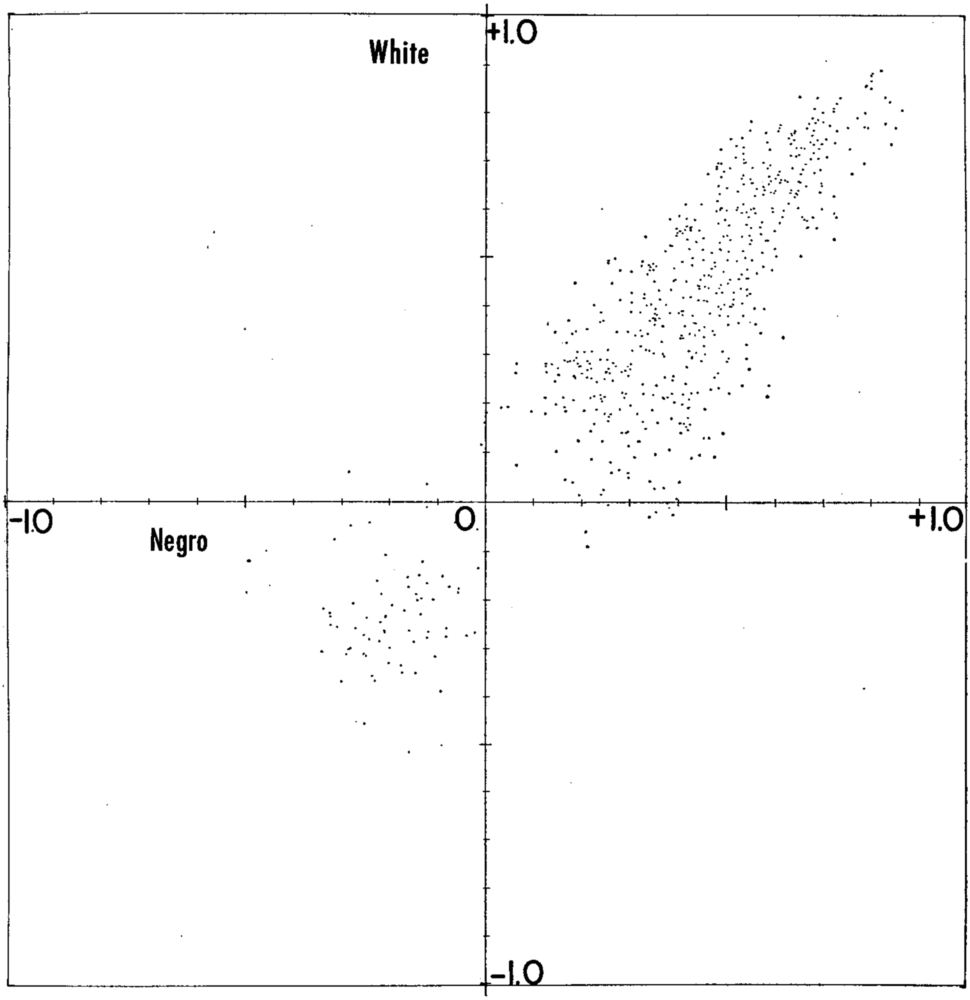


FIGURE 13
Scatter Plot of Coefficients of Correlation Between Reference-Test Scores for Negro Pupils (X Axis) and White Pupils (Y Axis). (Each point represents the correlation between a given pair of reference variables based on a sample of Negro pupils and a sample of white pupils.)

of the fourteenth matrix of residuals discouraged further factoring, but a centroid of curvature parameters was extracted to make certain that any potential learning-ability dimension would be included in the test space.

Since unity had been placed in the diagonal of the original matrix, the amount that the diagonal of the final residual matrix had been reduced indicated the communality that had been found. The communality was

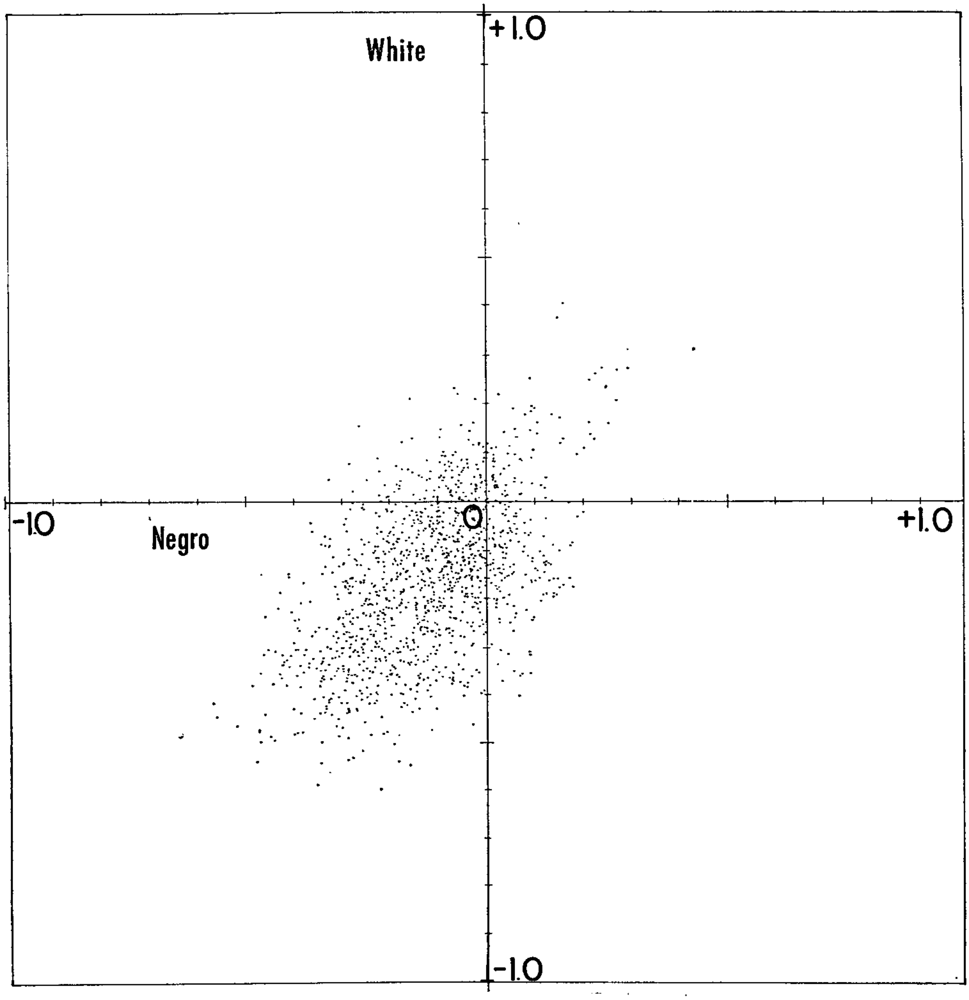


FIGURE 14
Scatter Plot of Coefficients of Correlation Between Learning Parameters and Reference-Test Scores for Negro Pupils (X Axis) and White Pupils (Y Axis). (Each point represents the correlation between a given learning variable and reference variable based on a sample of Negro pupils and a sample of white pupils.)

subject to improvement, of course, by more precise weighting of the group variables. Therefore, this derived communality was placed in the diagonal and the factor matrix was iterated until the input and output diagonals converged. Such iteration is standard procedure. The resulting factor matrix and the distribution of residuals based upon this final factor matrix appear in Appendix D.

Correlations between A and J parameters within a task were high. Since it seemed desirable not to introduce task specificity into the factor matrix no two parameters from the same task were used in any single group. The high correlations between parameters were reduced to the smaller communality of the two variables in the iterative procedure just mentioned. The variance specific to a task then remained outside the factor space. Correlations of the experimentally dependent variables have been excluded from the distribution of final residuals in Appendix D.

The iterated factor matrix was rotated obliquely to approach simple structure. The complete final factor matrix and the matrix of correlations between factors are presented in Appendix D.

CHAPTER V

RESULTS

It can be said that there are two objectives in the investigation of psychological variables. The more elementary objective is classification and identification. Exploratory research is successful if the existence of hypothesized variables or factors can be verified by analysis of the data. The second purpose of such investigations is the determination of the influence of one variable upon another. It is only the variables that are substantially correlated that are predictable. If an event is to be predicted, if the phenomenon is to be understood, and if a scientific law is to be formulated there must be both classification of variables and knowledge of their interdependence.

Both objectives have been kept in mind in this study. The factor analysis isolated fourteen factors which were rotated to the following interpretation: one race factor, two scholastic achievement factors, six reference factors, four learning factors, and one undefined factor. The findings will be reported in three parts. First there will be a discussion of what was revealed directly by the intercorrelations of the learning parameters, then a discussion of the reference and achievement factors, and last, a discussion of the learning factors.

Parameters of the Learning Tasks

The correlations between all variables are presented in Tables 2, 3, and 4 in Appendix C. Perhaps the most striking information to be obtained from Table 2, Intercorrelations of Learning Tasks, is the finding that the asymptote and curvature parameters for a given task are closely related. These correlations tend to be around +.80; for all tasks they exceed +.47. This means, of course, that the ranking of individual subjects changed very little from trial to trial during a given learning task. It means that few of the cumulative curves for different individuals on a given task intersect after leaving a common origin. The initial slope, the rate of curvature, and the level of the asymptote have yielded common information. (The factor structure verified this high interdependence of asymptote and curvature parameters for a given task.) In short, the obtained learning curves themselves are essentially one-independent-parameter, not two-independent-parameter, functions.

The fit parameter, which is a characteristic of the performance rather than of the hypothesized learning function, correlates both positively and negatively with the other parameters of the same task, from −.35 to +.45.

From Tables 2, 3, and 4 came Table 1, in which the correlations of the learning parameters with selected variables are summarized. It is clear from Table 2 that the relationships between learning parameters and scholastic aptitude and achievement scores are low in most cases, but are consistently positive. The pattern of correlations is remarkably uniform: for any parameter on any task the correlations with IQ and with achievement are about equal, the correlation with course marks tends to be somewhat lower, and the correlation with the same parameter on other tasks tends to be still lower. The asymptote parameter appears to be more closely related to aptitude and to be a better predictor of achievement than either the curvature or fit parameter. Only in parallel Tasks 6 and 10, (Listening I and II), was the fit parameter sufficiently related to aptitude and achievement scores to be considered a possible predictor. Why is a correlation of +.30 exceeded here and not in other tasks? It is reasonable to assume that

TABLE 1

Median Correlations of Learning Parameters on Each Learning Task With the Same Parameter on Other Tasks, With IQ's, With Achievement-Test Scores, and With Course Marks*.

	Asymptote Parameter of Task Number:											
Correlations with:	1	2	3	4	5	6	7	8	9	10	11	12
Other task asymptotes	.3	.1	.1	.2	.3	.2	.2	.2	.3	.2	.3	.3
IQ	.5	.3	.1	.3	.4	.5	.3	.3	.6	.4	.4	.5
Achievement	.5	.3	.1	.3	.4	.5	.4	.3	.6	.4	.5	.5
Marks	.4	.2	.1	.2	.3	.4	.3	.3	.5	.2	.4	.4
	Curvature Parameter of Task Number:											
Correlation with:	1	2	3	4	5	6	7	8	9	10	11	12
Other task curvatures	.1	.0	.1	.0	.1	.0	.1	.2	.2	.0	.1	.2
IQ	.4	.2	.1	.1	.3	.1	.3	.3	.4	.1	.2	.4
Achievement	.4	.2	.1	.1	.3	.1	.2	.3	.4	.1	.2	.4
Marks	.3	.1	.1	.1	.2	.1	.2	.2	.3	.0	.3	.3
	Fit Parameter of Task Number:											
Correlation with:	1	2	3	4	5	6	7	8	9	10	11	12
Other task fits	.1	.0	.0	.1	.0	.1	.0	.0	.1	.0	.1	.1
IQ	.2	.2	.0	.1	.0	.3	.1	.1	.1	.3	.1	.2
Achievement	.2	.2	.0	.1	.0	.3	.0	.1	.1	.3	.1	.2
Marks	.1	.1	.0	.1	.0	.1	.0	.2	.0	.3	.1	.2

*The parameter scales were inverted to give the sign of the correlation its usual connotation.

it was easier for a subject to recall his previous responses on all of the items of Tasks 6 and 10 than on the items of the other tasks. Those pupils who performed regularly on these two tasks tended to make exactly the same responses, not merely an equivalent number of right responses, on each trial. The difference between a regular performance here and on the other tasks, then, might be that in Tasks 6 and 10 the pupil has more control over his consistency. His inclination to be consistent and his satisfaction with previous responses would be revealed in his fit parameter. If this was the case the relevant correlations of Table 1 can be taken to mean that this inclination to be consistent is a more important correlate of aptitude than the actual regularity of performance. This is, of course, a tentative conclusion and needs further study.

Reference and Achievement Factors

The final rotated factor matrix is presented in Table 9 of Appendix D. The first factor of that matrix pertains to the racial dichotomy. The amount that a variable is correlated with race according to these data is its loading on the racial factor. Those partial relationships that are completely independent of race determine the structure that is represented by loadings for the other thirteen factors. The eight reference and achievement factors will be discussed next.

The number, vocabulary, and perceptual-speed factors were found to be defined quite well by the respective reference tests. The first trial score of Task 5, Word Memory I, has a loading on the vocabulary factor, but other variables associated with the learning tasks do not have appreciable loadings on any of these three factors.

The anticipated space-factor was found to be measured by the PMA nonverbal reasoning test as well as the two space tests. Therefore, this factor has been interpreted as a nonverbal reasoning factor. (Spatial ability is felt to be one sometimes-unique aspect of nonverbal reasoning ability.) The learning-curve parameters of Task 3, Jungle Maze; Task 7, Figure-Shape Matching; Task 8, Number Pattern I; and Task 11, Picture-Number Matching, have slight loadings on this factor.

A reasoning factor defined by the four reference reasoning tests was not found, but a factor that did appear loads most on the PMA verbal reasoning test and the asymptote of Task 9, Word Memory II. A group of variables with the next highest loadings on this factor depended on reading comprehension, but the Stanford Achievement reading-comprehension test was not in this group. The factor was labelled "verbal reasoning" since such an interpretation, although not altogether satisfactory, seemed better than any other.

The rote-memory factor was clearly defined by the reference-battery rote-memory tests (variables 49 and 50). It had been anticipated that the first trial scores for Tasks 5 and 11 (variable 47 from Word Memory I and variable 48 from Picture-Number Matching) would have substantial loadings on this factor since the four, as tests given once, are similar. However, they did not cluster together in the common-factor space. A possible important difference between the two pairs was the way in which the stimuli were exposed. For 47 and 48 the stimuli were presented sequentially at a predetermined rate. For 49 and 50 the subjects were able to distribute their study as they pleased during the interval allowed. By the end of the interval the subject presumably had examined the stimuli several times. His test score then might be more like an asymptote parameter than like a score for single-exposure memorization. The loadings for the asymptote parameters of Tasks 5 and 11 are, in fact, higher on this factor than the loadings of the corresponding first-trial scores, variables 47 and 48. The rote-memory factor here seems to be related to tasks on which the subject can examine the stimuli several times. Whether or not there is a separate memory factor for single exposures of stimuli is beyond the scope of this analysis.

The Otis Intelligence Test and the eight subtests of the Stanford Achievement Battery were found to define a factor which was named the achievement-scores factor. (The median achievement score correlated +.6 with race. Had oblique rotation toward race been permitted the achievement test loadings on this achievement scores factor would have been higher.) Learning Task 2, Word Groups, is the only learning task with noteworthy loadings on this factor.

The marks factor was particularly well defined. Course marks had small loadings on race, negligible loadings on all but the marks factor. The fact that course marks were found to be very compact in the common-factor space may be at least partly due to the fact that teachers generally assign all marks at a time, presumably while in one particular frame of mind. It was encouraging to find that the correlation between the achievement-scores factor and the course-marks factor is high enough (+.8) to mean that course marks reflect considerably more than a teacher's frame of mind.

Other coefficients of correlation between factors are given in Table 10. It is to be remembered that interdependent factors can be expected from oblique rotation. The interdependence of the reference factors here is high. Even between such seemingly different factors as perceptual speed and verbal reasoning the correlation is considerable (+.3); but a high interdependence of scores is not surprising when such a very heterogeneous group is tested. The interdependence of racial, reference, and achievement factors should not be overlooked even though the major attention here is on the differences between factors or abilities.

In summary, one dimension was defined by race. Rotation of the space orthogonal to race revealed eight non-learning dimensions which were interpreted as: number, vocabulary, perceptual speed, nonverbal reasoning, verbal reasoning, rote memory, achievement scores, and course marks. In general, the parameters of the learning tasks had small loadings on these reference factors.

Learning Factors

The high loadings on Factors VIII, XI, XII, XIII, and XIV of Table 9, Appendix D, indicated that they were learning factors. For the most part, non-learning-task variables had negligible loadings on these factors.

Factor VIII was a bipolar factor with an unstructured assortment of variables represented on both sides of the origin. Curvature and asymptote parameters were positive for the nonverbal relational tasks and the verbal rote-memory tasks, negative for the verbal relational tasks. The rote-memory fit parameters tended to be negative and the relational fit parameters positive. The most successful attempt at interpretation grouped the tasks needing a restrained, holistic, and reflective approach at one extreme, those with an impetuous but elemental approach at the other extreme. However, since such an interpretation was wholly speculative, based neither on the experimental design nor on actual observation, and since it violated common sense to assign some of the higher loaded variables to this hypothetical dimension, the factor was left uninterpreted.

Factor XI was found to be defined quite nicely by four fit parameters: those from Tasks 4, 5, 6, and 9—all with negative loadings. Except that Tasks 5 and 9 are parallel, the four are quite dissimilar. Observation during administration provided a clue for interpretation, however. Tasks 5 and 9, Word Memory I and II, were the final tasks of lengthy sessions. The children were fatigued. Some were noticeably inattentive. Task 6, Listening I, was long and boring, as was parallel Task 10 which had been given earlier. On Task 10 there was some expectation that the psychometrist would change the story on later trials. By the time Task 6 was administered there was general understanding that nothing new could be expected. On Task 4, Choice Board II, which was the first task which depended on the high-incentive apparatuses, some pupils were obviously more concerned about how the apparatus worked than in learning the correct associations. Each of these four tasks permitted or even encouraged a difference between subjects who concentrated on the learning task at hand and those who were distracted. This factor has been interpreted as a sustained-concentration factor. If this is a proper interpretation, the subjects whose scores are affected most by distractions in these circumstances tend to be slightly better performers on achievement tests. The correlation between the concentration factor and the achievement-scores factor is $-.2$.

The outstanding loadings on Factor XII are those for the fit parameters of Tasks 1, 7, and 11. Somewhat lower and of opposite sign are the curvature and asymptote parameters for Tasks 5, 7, 8, and 11. Since the oddnumbered tasks are the rote-memory tasks, this factor appears to be related to memory. Task 8, Number Pattern I, is not particularly out of place because it requires memorization. Tasks 1, 7, and 11 are alike in that they require matching of conventionally unrelated stimuli or, in other words, the synthesis of pairs from two small sets of stimuli. The stimuli need not be recalled. These three tasks differ with regard to the type of stimuli used, the method of presentation, and the method of testing. Other memory tasks differ from these in that then the learner is instructed to recall or to select a particular, conventionally unrelated subset of stimuli from a large but familiar set. There may be a factorial difference between recall or recognition or between pairing and forming a single group, but the differences here are not clear enough to set the interpretation along those lines. Factor XII is a memory learning factor that is primarily defined by the regularity of performance on three tasks involving memory of paired stimuli. For the purposes of this study it will suffice to consider this factor simply as a memory learning factor.

Factor XIII appears to be determined by the content of the material to be learned. The predominant variables on this factor were the curvature and asymptote parameters of Tasks 4, 8, 11, and 12. All these tasks and only these tasks depend on the use of numbers, so this factor is interpreted as a numerical-task learning factor. The correlation between this factor and the reference battery number factor is +.3.

The structure of Factor XIV is good but the interpretation is somewhat questionable. The highest loadings are associated with the curvature and asymptote parameters of Tasks 1, 3, 5, and 9, all rote memory tasks, and the fit parameter of Task 8, a relational learning task. If the high loading (+.5) of the latter parameter can be ignored, the factor is clearly related to memory. The loadings of the other variables seemed to warrant this step, and this factor has been designated a second memory-task learning factor.

The asymptote and curvature parameters of all the memory tasks have loadings on one or the other memory factor. The major difference between these two factors is that XII is defined by three fit parameters and XIV is defined by several asymptote and curvature parameters. The correlation between the two memory learning factors is almost zero. Neither of them is related to the rote-memory factor of the reference battery.

In summary, there were five factors that were determined by parameters of the learning tasks. Two were related to rote-memory tasks, one was specific to tasks in which numbers were manipulated, one was seen to be related to the regularity of performances under distracting circumstances, and the other factor was not interpreted. Except for a correlation of +.3 between the numerical-task learning factor and one of the memory learning factors, Factor XIV, the intercorrelation of learning factors is negligible.

The numerical-task learning factor has low positive correlations with reference factors; the other learning factors appear to have zero or low negative correlation with reference and achievement factors. There is very little, if any, evidence here that the obtained learning factors are related to achievement as measured by course marks.

The correlations of Table 1 might cause one to infer that the asymptote parameters would be most definitive in the determination of factors and the fit parameters least definitive. As for what was defined by rotation to simple structure, the opposite was true. The fit parameters determined two of the factors. On the other two interpreted learning factors the curvature parameters generally had higher loadings than the asymptote parameters. According to this it would be proper to conclude that the asymptote parameter is probably a better index of scholastic achievement than the curvature parameter, but the curvature parameter is a better index of certain learning factors than the asymptote.

Conclusions

The use of the theoretical hyperbola for these learning data was judged to be highly satisfactory. Although other curve families might have provided an equally good fit, it is believed that they would not have given better fits. A check of the signs of the deviations from the fitted curve indicated that there was no systematic departure (such as too much curvature initially, too little later) from the plots. The fit parameter, then, was properly considered a regularity-of-performance parameter, not an appropriateness-of-function parameter.

It is of some importance to note that a single mathematical equation proved to fit data from twelve different learning tasks on which the performances ranged from random to "completely insightful." Different parameters gave all the flexibility necessary for good fits. Different equations were not needed. It follows that difference in curve shape is not necessarily evidence to support the hypothesis that there is more than one learning function.

The data of this experiment support a one-factor theory of learning, if one-factor means that one mathematical equation is sufficient to represent learning in general. If, on the other hand, one-factor means that learning in one learning situation is equivalent to learning in another, there is contrary evidence here. Learning ability does not appear to be a unitary trait. On some tasks one group of subjects will be the better learners, on certain other tasks they will be inferior. This conclusion is based upon the fact that this factor analysis and rotation revealed four different interpretable learning factors.

The interpretation of the learning factors was not as rigorous as it would have been if the tasks had been designed to vary in fewer ways. But an exploratory study of learning parameters would have been hampered, it is believed, by focus on one particular task, such as serial learning of verbal stimuli. The intent here was to devise tasks which represent short-term, scholastic learning experiences. Common findings on different tasks permit generalizations that could not come from the highly focused study. As it stands, learning factors, unrelated to factors defined by tests given just once, have been found. More precise interpretation than has been given here will have to come from subsequent investigation.

With particular reference to the hypotheses listed in the Introduction, the findings of this research support these conclusions:

1. Learning-curve parameters are significantly correlated with measured intelligence, aptitudes, and achievements. The degree of correlation is substantial enough to support the conclusion that characteristics of some short-term learning performances can be predicted from intelligence-test scores.

2. Upon factor analysis, learning-curve parameters load on some factors which are independent of factors determined by tests given just once. The extracted learning factors did not, however, contribute an appreciable amount, quantitatively, to the explanation of the total variability between individuals.

3. Learning factors and parameters of the type studied here do not correlate with academic course marks when scholastic aptitude, as conventionally measured, is controlled. The correlation between the marks factor and the learning factors is negligible.

4. For widely different tasks there is not a general learning ability except that which can be measured by conventional tests given only once.

5. For a given performance of the type studied here there is a high correlation between the asymptote or total errors parameter and the curvature or learning-ability parameter. The subject whose curve indicates superior performance at the outset of learning is most likely to appear "insightful" and to commit the fewest total errors. The fit or regularity parameter is sometimes moderately correlated with the other two parameters either positively or negatively.

6. Learning-curve parameters from numerical tasks differentiated between individuals sufficiently to determine a learning factor. Thus, a learning factor can be defined by the content of material to be learned. It was not found that verbal tasks load on a verbal learning factor nor that nonverbal tasks load on a nonverbal factor.

7. No factors were found to support the hypothesis that a rote learning performance is fundamentally different from a relational learning performance.

8. Learning-curve parameters were not found to load on factors which are related to the degree to which the experimental circumstances raise the level of motivation. Added incentive had a universal rather than an indi-

vidual effect: the superior learners excelled and the non-learners failed to learn regardless of the incentive provided.

Recommendations

There is much more information in the data collected for this study than was analyzed for this monograph. For example, a full report of the racial differences in this learning area could be and should be compiled. One particular aspect of these learning-parameter data which should be studied at greater length is the relative value of the various parameters for describing the performance. It was found that the asymptote and curvature parameters are highly correlated, yet the asymptote seems to be more indicative of the child's aptitude and the curvature seems to be more indicative of some of the learning factors. Perhaps other indices can be found. Perhaps scores on one or a few trials can provide the same information more economically. With these questions in mind, further analyses of the collected data should be undertaken.

It is recommended that the course marks for the children in this sample be collected again after several years have passed to verify the conclusion (based on correlations with contemporary course marks) that short-term learning parameters are unrelated to such achievements.

The fact that parameters of short-term learning were not found to be highly correlated with academic course marks does not mean that they might not be put to good advantage as an alternative measure of scholastic aptitude. (As devised for this study, the tasks were not "culture-free.") Tasks that are little concerned with advanced symbolism, that involve responses which in themselves are not a separate task for the learners, and that utilize non-scholastic materials familiar to the subject might do a better job of measuring capacity for learning than the tests that have been developed and distributed as "culture-free" tests.

In this study factors have again been found to be related to the type of experimental task used. Existence of multiple factors means that there are important, consistent differences among learners. It is recommended that more emphasis be placed on factorial determination of the equivalence of the many experimental learning situations in contemporary use.

To some readers the most important question at this time is: How important are the conclusions reached here for learning which continues over a much longer period, such as the learning of a new language? Generalizations from this study to long-term learning are questionable. The asymptote and curvature parameters might be completely uncorrelated in that case. The interrelation of learning ability and achievement might be found to be much higher than it was in this study. The final recommendation is that research should be undertaken to investigate the same relationships in longer-term learning situations.

CHAPTER VI

SUMMARY

This research project was an investigation of the individual differences in certain learning performances with particular reference to measures of various mental abilities and achievements. According to the literature on human learning it had not been ascertained whether a general learning ability factor could be identified in all learning performances, whether there are multiple learning abilities that are specific to each task or type of task, or even whether learning ability could be defined by the results of tests given once only.

To investigate these relationships a dozen learning tasks were devised. These tasks varied as to their verbal or nonverbal content, as to whether rote or relational learning was required, and as to the incentive that was provided. They were selected to parallel some common scholastic learning experiences. A reference battery of intelligence, achievement and factorial aptitude tests was assembled.

Learning task scores, reference measurements, and course marks were obtained from 240 children. The data for each learning performance were fitted by a theoretical curve, (a modification of a rational hyperbola derived by Thurstone). Three parameters, an asymptotic or total errors parameter, a curvature or learning ability parameter, and a goodness of fit or regularity of performance parameter, were obtained for each performance. These parameters, the reference measurements, and the course marks were intercorrelated. Correlations of the goodness of fit parameters with aptitude and achievement and even the intercorrelations of fit parameters on different tasks were found generally to be negligible. It was found that the curvature and asymptote parameters were substantially correlated, +.1 to +.6, with scholastic aptitude and achievement as measured by conventional standardized tests. Thus, unlike the majority of previous studies, there is support here for defining intelligence as the ability to learn. Correlation of the curvature and asymptote parameters with course marks ranged from 0 to +.5. Intercorrelations of curvature and asymptote parameters for the same task were very high. This means that those learners who start well finish well. There could have been only a few subjects, if there were any, who "start slowly and finish fast" on this kind of task.

The matrix of correlations was factor analyzed. The final rotated factor matrix yielded one racial factor, two achievement factors, six reference factors, four learning factors and one uninterpreted factor. Two of the learning factors were interpreted as memory-task learning factors. These factors were independent of the rote memory factor obtained from reference tests. Another learning factor was found to be associated with tasks which required considerable use of numbers, so it was interpreted as a numerical-task learning factor. The fourth learning factor was defined primarily by the goodness of fit parameter for four tasks. These four tasks, by being boring or distracting, seemed to require a special effort on the part of the learner to perform regularly and successfully. This factor has been interpreted as a concentration factor.

The findings of this study revealed no general learning ability other than the general aptitude that is measured by such tests as an intelligence test given just once. Added incentive, one of the design variables, had universal rather than individual effect: the same learners excelled regardless of the incentive provided. The association of one of the learning factors with a group of numerical tasks supports the hypothesis that learning ability can be specific to a type of task. No factors were found to support the hypothesis that a rote learning performance is fundamentally different from a relational learning performance.

APPENDIX A

PARTICIPATING SCHOOLS

Pretesting

Littlebrook School, Princeton, New Jersey; Mr. William Purcell, Principal.
West Windsor School, Dutch Neck, New Jersey; Mr. James C. Sandilos, Superintendent.
Jefferson School, Trenton, New Jersey; Miss Gloria Fried, Principal.
Georgia Avenue School, Atlanta; Mr. John Y. Moreland, Principal.
Moses W. Formwalt School, Atlanta; Mr. Wesley H. Cook, Principal.
R. L. Hope School, Atlanta; Miss Miriam Riley, Principal.

Testing

Atlanta Public Schools, Atlanta, Georgia; Miss Ira Jarrell, Superintendent; Mr. Gordon Fort, Director of Testing.
- Morris Brandon School; Mrs. Zerah S. Baggett, Principal.
- E. R. Carter School; Mrs. Florine Furlow, Principal.
- Capitol View School; Miss Marion Jack, Principal.
- Lena Jean Campbell School; Mr. Earl A. Starling, Principal.
- E. P. Howell School; Mrs. Mabelle Pickert, Principal.
- Thomas H. Slater School; Mr. Andrew Jackson Lewis II, Principal.
- Perkerson School; Miss Sarah Evelyn Smith, Principal.
- E. A. Ware School; Mr. Otis White, Principal.

APPENDIX B

TESTING SCHEDULE

Session	Duration of Single Admin.	Battery	Tests Included Var. No.	Title
I	70 min.	Reference	64	Attention
			49	Rote Memory
			51	Reasoning
			59	Perceptual
			54	Reasoning
			61	Space
			58	Perceptual
			50	Rote Memory
			60	Perceptual
			75	Persistence
II	90 min.	Reference	73	Attention
			46	Otis
			65	PMA Verbal W
			66	PMA Verbal P
			62	PMA Space
			52	PMA Reasoning W
			53	PMA Reasoning F
			57	PMA Perceptual
			55	PMA Number
			74	Persistence
III	90 min.	Learning Tasks		Tasks 12–9
IV	75 min.	Learning Tasks		Tasks 8–5
V	75 min.	Learning Tasks		Tasks 4–1

APPENDIX C

CORRELATION MATRICES

TABLE 2

Intercorrelations of Learning Tasks

		VAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14
TASK 1	ERRORS	01														
	CURVE	02	.47													
	FIT	03	.15	.19												
TASK 2	ERRORS	04	.15	.10	-.02											
	CURVE	05	.12	.07	-.00	.98										
	FIT	06	.09	.14	-.01	.15	.09									
TASK 3	ERRORS	07	.21	.12	.10	.09	.08	-.03								
	CURVE	08	.21	.11	.10	.07	.07	-.03	.95							
	FIT	09	.10	.13	.02	.06	.09	-.04	.45	.44						
TASK 4	ERRORS	10	.29	.11	.12	.09	.08	-.03	.14	.14	.07					
	CURVE	11	-.00	.02	.01	-.02	-.02	-.03	.03	.03	.00	.71				
	FIT	12	.13	.04	.03	.13	.13	-.04	.07	.04	-.05	-.12	-.35			
TASK 5	ERRORS	13	.36	.23	.02	.11	.07	.11	.23	.19	.01	.11	.04	-.01		
	CURVE	14	.25	.15	.02	.08	.04	.02	.18	.14	.01	.09	.06	-.08	.89	
	FIT	15	-.00	-.03	.02	-.05	-.06	.06	.00	.03	.08	.04	.02	.04	-.22	-.20
TASK 6	ERRORS	16	.32	.24	.08	.08	.04	.24	.09	.10	-.02	.27	.09	.07	.30	.25
	CURVE	17	.04	.10	-.01	-.06	-.07	.16	-.03	-.01	.01	.07	-.00	-.02	.01	.02
	FIT	18	.14	.03	.08	.08	.07	.07	-.02	-.01	.07	.07	-.06	.16	.13	.10
TASK 7	ERRORS	19	.29	.14	.06	.16	.13	.08	.07	.07	.12	.21	.03	-.02	.25	.19
	CURVE	20	.26	.09	-.02	.14	.11	.04	-.01	-.01	.07	.16	-.01	-.03	.24	.20
	FIT	21	-.00	-.01	.14	-.04	-.02	.09	.12	.11	.02	.06	.09	.01	-.01	-.00
TASK 8	ERRORS	22	.26	.20	.06	.06	.03	.02	.18	.18	.04	.23	.11	-.03	.27	.25
	CURVE	23	.23	.17	.11	.13	.11	.03	.14	.14	.06	.22	.10	.03	.25	.20
	FIT	24	.11	.07	.04	.05	.04	.13	.06	.04	.03	.05	.03	-.05	.23	.08
TASK 9	ERRORS	25	.42	.29	.07	.16	.12	.14	.13	.09	-.03	.17	-.01	.01	.66	.48
	CURVE	26	.17	.24	.06	.17	.14	.15	.13	.11	.04	.13	.03	-.11	.42	.40
	FIT	27	.15	-.06	-.06	.10	.08	.04	.06	.07	-.01	-.01	-.10	.31	.14	.16
TASK 10	ERRORS	28	.22	.13	.09	.12	.09	.05	.20	.17	-.02	.13	-.01	.10	.12	.10
	CURVE	29	.04	.01	.04	.02	.01	-.04	.10	.09	-.00	-.02	-.06	-.01	-.01	-.01
	FIT	30	.25	.10	-.01	.08	.04	.02	-.02	-.03	.06	.24	.12	-.01	.30	.31
TASK 11	ERRORS	31	.28	.21	.10	.04	.02	.07	.06	.03	.10	.19	.09	-.06	.31	.25
	CURVE	32	.09	.10	.05	.01	.00	.02	.03	.00	.06	.03	.01	-.06	.18	.13
	FIT	33	.07	-.00	.30	.02	.03	-.06	.00	-.01	.14	.14	.11	.05	-.01	.01
TASK 12	ERRORS	34	.32	.27	.18	.12	.10	.08	.18	.15	.00	.36	.17	.01	.33	.27
	CURVE	35	.29	.25	.15	.07	.04	.10	.19	.17	.03	.31	.18	.01	.28	.23
	FIT	36	.30	.13	.11	.14	.15	.02	.34	.36	.03	.25	.03	.14	.25	.22

TABLE 2 (Continued)

15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
.03																					
-.00	.75																				
.10	.01	-.24																			
-.03	.23	.00	.20																		
-.05	.14	-.04	.17	.86																	
.03	.05	.10	-.00	-.02	-.22																
-.04	.17	-.02	.09	.22	.19	.01															
-.07	.18	-.01	.08	.24	.20	.01	.83														
.02	.00	-.08	.02	-.00	.00	-.07	.36	.30													
-.11	.37	.02	.13	.28	.23	-.03	.27	.22	.23												
-.06	.25	.04	.10	.23	.20	.02	.26	.21	.04	.74											
.15	.03	-.01	.13	.08	.08	.06	.00	.05	-.11	-.11	-.15										
.04	.27	.05	.10	.22	.16	.19	.09	.12	-.07	.22	.18	.09									
.03	.10	.04	-.04	.11	.10	-.15	-.02	.01	-.08	.02	.04	-.02	.84								
.05	.27	.14	.08	.15	.10	-.02	.10	.09	.09	.33	.20	.08	.07	-.11							
-.06	.24	.06	.24	.37	.26	.06	.37	.33	.15	.39	.27	.02	.11	-.06	.22						
-.07	.13	.06	.17	.22	.14	.01	.27	.21	.14	.24	.16	-.03	.02	-.07	.06	.86					
-.03	.07	.02	.07	.01	-.07	.19	-.01	.03	-.03	-.02	-.05	.04	.13	.08	.08	-.07	-.31				
-.05	.25	.04	.08	.26	.21	.12	.38	.36	.13	.36	.23	.07	.24	.09	.17	.47	.25	.23			
-.02	.25	.09	.06	.26	.21	.10	.38	.40	.16	.28	.19	.07	.16	.03	.12	.40	.22	.19	.86		
-.08	.17	.04	-.01	.16	.10	.05	.09	.10	-.00	.25	.22	.06	.09	-.04	.14	.16	.04	.14	.44	.38	

TABLE 3

Intercorrelations of Learning Tasks with Reference Variables

		VAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
STANFORD ACH	RDG CMPN	37	-.44	-.31	-.19	-.27	-.22	-.15	-.13	-.10	.03	-.33	-.13	-.11	-.46	-.38	.02
	VOCAB	38	-.44	-.35	-.18	-.31	-.27	-.14	-.09	-.08	.07	-.29	-.12	-.13	-.40	-.30	.03
	SPELLING	39	-.38	-.40	-.15	-.26	-.22	-.14	-.06	-.05	.04	-.23	-.11	-.07	-.42	-.30	-.00
	ARITH REAS	40	-.46	-.31	-.19	-.26	-.22	-.19	-.13	-.11	.03	-.34	-.11	-.12	-.43	-.34	.00
	COMPUTN	41	-.44	-.29	-.15	-.28	-.23	-.18	-.14	-.11	-.02	-.33	-.13	-.11	-.44	-.36	-.03
	SOCIAL ST	42	-.43	-.29	-.20	-.28	-.25	-.09	-.09	-.09	.03	-.33	-.11	-.12	-.37	-.29	.04
	SCIENCE	43	-.44	-.33	-.18	-.31	-.27	-.16	-.06	-.06	.02	-.26	-.06	-.14	-.33	-.24	.00
	STUDY SK	44	-.51	-.35	-.21	-.28	-.24	-.14	-.14	-.12	.01	-.36	-.09	-.17	-.40	-.31	-.01
	MD GRADE EQ	45	-.47	-.37	-.21	-.32	-.28	-.17	-.12	-.11	.03	-.33	-.11	-.14	-.40	-.32	-.01
OTIS	IQ	46	-.47	-.37	-.17	-.29	-.24	-.16	-.13	-.12	.00	-.34	-.11	-.09	-.37	-.27	-.02
ROTE MEMORY	TASK 5	47	.22	.17	.07	.05	.03	-.03	.08	.07	.02	.09	-.00	.12	.28	-.02	.04
	TASK 11	48	.27	.20	.07	.06	.04	.14	.03	.01	.09	.25	.12	-.01	.18	.14	.02
	REF B	49	-.22	-.20	-.10	-.14	-.14	-.02	-.09	-.10	-.06	-.18	-.12	-.01	-.31	-.28	-.02
	REF H	50	-.35	-.27	-.12	-.18	-.15	-.06	-.09	-.07	-.04	-.21	-.02	-.08	-.46	-.36	-.08
REASON	REF C	51	-.43	-.31	-.17	-.28	-.25	-.13	-.13	-.15	-.02	-.32	-.10	-.14	-.30	-.24	-.03
	PMA RW	52	-.48	-.35	-.11	-.21	-.16	-.15	-.14	-.11	-.02	-.26	-.08	-.08	-.50	-.37	.05
	PMA RF	53	-.32	-.19	-.12	-.18	-.16	-.14	-.29	-.27	-.14	-.31	-.12	-.09	-.27	-.20	.08
ARI REAS	REF E	54	-.51	-.35	-.14	-.23	-.19	-.11	-.15	-.13	.01	-.36	-.10	-.16	-.40	-.32	-.09
NUMBER	PMA N	55	-.41	-.30	-.14	-.17	-.13	-.09	-.22	-.18	.02	-.34	-.13	-.00	-.42	-.29	-.01
	PST 1	56	-.42	-.30	-.20	-.19	-.15	-.07	-.10	-.08	.10	-.26	-.11	-.07	-.46	-.36	.00
PERCEPT	PMA P	57	-.28	-.09	-.05	-.14	-.13	-.14	-.12	-.13	-.04	-.19	.01	-.01	-.23	-.11	.06
	REF G	58	-.24	-.17	-.02	-.03	-.01	-.02	-.08	-.08	.04	-.21	-.14	-.03	-.25	-.19	.02
	REF D	59	-.27	-.26	-.08	-.17	-.13	-.03	-.08	-.07	.02	-.11	.04	.00	-.24	-.16	.14
	REF I	60	-.36	-.19	-.07	-.12	-.08	-.07	-.21	-.18	.05	-.25	-.10	-.04	-.35	-.21	-.03
SPACE	REF F	61	-.17	-.09	-.06	-.03	-.04	.01	-.10	-.10	-.02	-.03	.09	.07	-.05	-.00	.06
	PMA S	62	-.40	-.22	-.16	-.19	-.17	-.17	-.22	-.21	-.15	-.29	-.05	.03	-.29	-.24	.07
RACE		63	-.40	-.25	-.16	-.26	-.20	-.12	-.12	-.10	.07	-.28	-.09	-.13	-.42	-.35	.00
WARM UP	REF A	64	-.45	-.32	-.08	-.25	-.20	-.12	-.11	-.07	.05	-.30	-.06	-.17	-.48	-.35	-.01
VERBAL	PMA VW	65	-.48	-.32	-.15	-.25	-.21	-.11	-.11	-.11	.03	-.31	-.09	-.15	-.46	-.32	-.04
	PMA VP	66	-.49	-.31	-.19	-.24	-.20	-.22	-.11	-.08	.02	-.33	-.10	-.12	-.28	-.19	.03
OTIS	RAW	67	-.50	-.38	-.18	-.31	-.26	-.18	-.14	-.12	-.02	-.36	-.10	-.12	-.39	-.29	-.04
GRADES	SPELLING	68	-.34	-.28	-.09	-.17	-.13	-.07	-.08	-.08	-.01	-.22	-.06	-.07	-.38	-.26	-.01
	ARITH	69	-.40	-.29	-.09	-.12	-.08	-.10	-.10	-.09	.03	-.25	-.11	-.05	-.35	-.25	-.00
	READING	70	-.38	-.25	-.04	-.21	-.17	-.08	-.03	-.06	.02	-.21	-.04	-.13	-.32	-.19	-.03
	LANG	71	-.35	-.25	-.04	-.15	-.11	-.06	-.12	-.13	-.05	-.23	-.05	-.06	-.35	-.22	-.02
	SOCIAL ST	72	-.34	-.26	-.06	-.15	-.12	-.11	-.03	-.04	.03	-.24	-.09	-.10	-.30	-.20	-.00
MEAN			3.03	4.38	1.42	6.07	7.04	1.77	3.96	5.34	2.01	4.30	5.55	4.95	4.42	6.61	1.40
SD			0.86	0.80	1.10	3.39	2.62	1.61	1.28	1.47	1.96	0.98	1.11	5.65	0.85	0.81	1.06

TABLE 3 (Continued)

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
-.46	-.03	-.26	-.37	-.30	-.04	-.32	-.30	-.06	-.54	-.42	-.10	-.38	-.11	-.27	-.44	-.28	-.09	-.45	-.41	-.20
-.45	-.05	-.22	-.32	-.26	-.03	-.25	-.23	-.03	-.51	-.39	-.11	-.38	-.07	-.26	-.37	-.21	-.10	-.39	-.35	-.21
-.45	-.10	-.12	-.22	-.16	.03	-.27	-.26	-.14	-.58	-.42	-.02	-.23	.03	-.29	-.37	-.21	-.08	-.40	-.39	-.22
-.42	-.04	-.27	-.35	-.30	-.03	-.35	-.34	-.13	-.54	-.43	-.15	-.36	-.07	-.30	-.44	-.24	-.08	-.49	-.46	-.23
-.43	-.04	-.25	-.33	-.25	-.07	-.32	-.33	-.08	-.55	-.40	-.11	-.32	-.04	-.32	-.45	-.23	-.14	-.54	-.46	-.26
-.43	-.02	-.31	-.40	-.34	.00	-.26	-.26	-.01	-.48	-.33	-.11	-.33	-.02	-.29	-.42	-.24	-.11	-.39	-.36	-.20
-.46	-.07	-.27	-.36	-.29	-.05	-.24	-.23	-.01	-.48	-.35	-.10	-.36	-.07	-.30	-.39	-.21	-.09	-.35	-.32	-.19
-.49	-.06	-.30	-.38	-.33	-.06	-.36	-.38	-.03	-.54	-.38	-.13	-.42	-.12	-.31	-.48	-.28	-.10	-.47	-.44	-.24
-.49	-.07	-.28	-.36	-.29	-.06	-.31	-.29	-.05	-.56	-.43	-.12	-.40	-.09	-.31	-.45	-.26	-.10	-.46	-.43	-.25
-.47	-.06	-.28	-.31	-.24	-.00	-.33	-.28	-.09	-.55	-.43	-.12	-.36	-.07	-.30	-.42	-.23	-.12	-.47	-.41	-.23
.20	.02	.02	.10	.05	.03	.09	.08	.16	.43	.17	-.10	.07	.01	.07	.21	.15	.05	.20	.18	.10
.21	.06	.07	.29	.23	.05	.19	.21	.04	.16	.07	.12	.11	-.01	.17	.32	-.06	.26	.31	.29	.12
-.19	-.00	-.14	-.24	-.13	-.14	-.20	-.20	-.02	-.30	-.26	-.08	-.14	-.00	-.20	-.43	-.24	-.14	-.43	-.35	-.19
-.37	-.11	-.17	-.30	-.23	-.01	-.31	-.27	-.13	-.53	-.38	-.10	-.19	.03	-.25	-.43	-.22	-.13	-.44	-.37	-.19
-.33	-.02	-.16	-.25	-.22	-.04	-.29	-.29	-.11	-.40	-.30	-.10	-.32	-.08	-.25	-.36	-.19	-.07	-.43	-.37	-.30
-.51	-.08	-.19	-.31	-.24	.07	-.32	-.31	-.14	-.69	-.46	-.03	-.33	-.07	-.34	-.42	-.23	-.08	-.40	-.36	-.23
-.33	-.03	-.22	-.33	-.24	-.13	-.43	-.39	-.10	-.40	-.34	-.06	-.30	-.12	-.14	-.38	-.18	-.16	-.40	-.35	-.24
-.45	-.04	-.19	-.31	-.25	.04	-.35	-.33	-.05	-.58	-.43	-.12	-.32	-.02	-.38	-.44	-.23	-.03	-.48	-.42	-.27
-.32	.02	-.15	-.18	-.14	.01	-.33	-.28	-.12	-.61	-.45	.01	-.24	-.03	-.23	-.38	-.20	-.03	-.48	-.37	-.25
-.30	.04	-.22	-.27	-.24	-.00	-.28	-.28	-.06	-.56	-.44	-.08	-.24	-.03	-.21	-.40	-.23	-.08	-.48	-.40	-.22
-.24	.02	-.10	-.21	-.17	-.04	-.28	-.24	-.10	-.39	-.17	.02	-.23	-.08	-.16	-.28	-.14	-.06	-.32	-.23	-.24
-.18	.02	-.03	-.18	-.17	-.01	-.18	-.21	.00	-.32	-.22	-.07	-.15	.00	-.02	-.25	-.10	-.04	-.32	-.27	-.21
-.26	-.08	.02	-.20	-.21	.02	-.22	-.17	-.11	-.35	-.25	-.03	-.15	-.03	-.19	-.23	-.09	-.09	-.40	-.29	-.27
-.26	.04	-.02	-.15	-.13	-.05	-.30	-.29	-.12	-.48	-.30	.02	-.15	-.01	-.09	-.33	-.16	-.01	-.46	-.37	-.23
-.07	-.04	-.06	-.13	-.07	-.03	-.20	-.13	-.10	-.15	-.17	.09	-.10	-.07	-.08	-.20	-.14	.02	-.23	-.23	-.10
-.32	-.04	-.24	-.38	-.30	.00	-.39	-.36	-.13	-.38	-.29	.03	-.25	-.07	-.21	-.39	-.19	-.07	-.42	-.37	-.29
-.33	.02	-.21	-.23	-.20	-.04	-.24	-.22	-.05	-.49	-.39	-.08	-.21	.03	-.23	-.24	-.08	-.07	-.32	-.26	-.17
-.45	-.01	-.20	-.28	-.24	.08	-.32	-.30	-.16	-.67	-.45	-.08	-.28	.00	-.30	-.39	-.19	-.08	-.42	-.34	-.22
-.52	-.08	-.24	-.36	-.30	-.01	-.30	-.28	-.08	-.61	-.42	-.12	-.36	-.02	-.28	-.42	-.24	-.11	-.45	-.39	-.30
-.42	.02	-.27	-.38	-.30	-.02	-.26	-.24	-.02	-.47	-.33	-.07	-.39	-.09	-.25	-.36	-.14	-.14	-.38	-.31	-.24
-.49	-.06	-.28	-.34	-.26	-.00	-.34	-.30	-.10	-.57	-.45	-.13	-.36	-.07	-.32	-.43	-.22	-.12	-.47	-.41	-.22
-.43	-.12	-.16	-.25	-.19	.05	-.26	-.24	-.16	-.57	-.41	.04	-.18	.05	-.29	-.45	-.34	-.06	-.35	-.33	-.23
-.41	-.10	-.13	-.26	-.20	-.00	-.33	-.31	-.18	-.50	-.32	.01	-.22	.01	-.33	-.49	-.34	-.06	-.41	-.38	-.22
-.44	-.08	-.20	-.29	-.24	.08	-.22	-.21	-.17	-.56	-.36	.01	-.26	.00	-.34	-.39	-.24	-.09	-.34	-.32	-.26
-.38	-.07	-.12	-.22	-.15	.03	-.26	-.23	-.18	-.52	-.32	.02	-.23	-.01	-.31	-.44	-.31	-.07	-.39	-.37	-.25
-.40	-.09	-.14	-.26	-.21	.04	-.23	-.24	-.14	-.45	-.29	.01	-.23	-.00	-.29	-.43	-.30	-.07	-.37	-.34	-.26
4.41	6.23	1.18	3.97	6.12	1.90	4.74	6.99	1.12	4.10	6.31	1.59	4.60	6.88	0.93	4.17	5.69	2.77	3.57	5.79	3.10
1.53	1.69	1.32	1.16	1.34	2.02	1.40	1.34	1.29	0.86	0.64	1.39	1.75	2.21	1.66	0.79	0.86	3.22	1.00	1.52	3.57

TABLE 4
Intercorrelations of Reference Variables

		VAR	37	38	39	40	41	42	43	44	45	46	47	48	49	50
STANFORD	RDG CMPN	37														
ACH	VOCAB	38	.84													
	SPELLING	39	.71	.75												
	ARITH REAS	40	.78	.80	.72											
	COMPUTN	41	.73	.73	.72	.79										
	SOCIAL ST	42	.81	.85	.69	.77	.69									
	SCIENCE	43	.79	.85	.71	.77	.70	.86								
	STUDY SK	44	.82	.81	.71	.82	.79	.84	.80							
	MD GRADE EQ	45	.88	.92	.81	.87	.82	.89	.90	.91						
OTIS	IQ	46	.83	.83	.76	.81	.78	.80	.80	.84	.89					
ROTE	TASK 5	47	-.25	-.28	-.35	-.19	-.24	-.21	-.24	-.24	-.25	-.27				
MEMORY	TASK 11	48	-.31	-.29	-.26	-.32	-.32	-.34	-.32	-.31	-.36	-.33	.09			
	REF B	49	.39	.37	.33	.39	.41	.33	.33	.40	.39	.35	-.20	-.25		
	REF H	50	.55	.54	.66	.56	.61	.51	.50	.57	.59	.60	-.32	-.28	.50	
REASON	REF C	51	.56	.57	.50	.60	.58	.57	.58	.63	.64	.64	-.20	-.33	.37	.43
	PMA RW	52	.73	.70	.73	.73	.74	.69	.67	.75	.75	.77	-.35	-.29	.30	.60
	PMA RF	53	.46	.45	.35	.49	.55	.46	.44	.54	.52	.54	-.17	-.30	.29	.37
ARI REAS	REF E	54	.75	.71	.68	.79	.75	.72	.71	.79	.78	.75	-.24	-.29	.39	.58
NUMBER	PMA N	55	.58	.55	.60	.62	.65	.50	.48	.59	.59	.60	-.34	-.22	.36	.53
	PST 1	56	.64	.65	.68	.69	.72	.60	.58	.64	.66	.67	-.32	-.24	.40	.55
PERCEPT	PMA P	57	.29	.25	.28	.29	.39	.29	.27	.35	.31	.38	-.21	-.17	.17	.30
	REF G	58	.31	.28	.33	.28	.35	.24	.20	.31	.26	.30	-.21	-.15	.25	.39
	REF D	59	.31	.31	.38	.31	.43	.26	.23	.35	.34	.45	-.17	-.15	.29	.37
	REF I	60	.46	.38	.48	.44	.52	.35	.30	.44	.42	.48	-.34	-.21	.32	.51
SPACE	REF F	61	.19	.22	.12	.23	.22	.20	.19	.21	.22	.24	-.06	-.07	.08	.09
	PMA S	62	.47	.43	.38	.56	.56	.48	.45	.55	.50	.53	-.10	-.28	.31	.37
RACE		63	.60	.61	.54	.66	.56	.58	.58	.61	.61	.55	-.20	-.18	.34	.48
WARM UP	REF A	64	.74	.70	.70	.72	.70	.71	.67	.73	.74	.76	-.35	-.27	.38	.64
VERBAL	PMA VW	65	.80	.82	.78	.76	.75	.81	.79	.80	.84	.83	-.38	-.29	.39	.63
	PMA VP	66	.69	.75	.56	.67	.65	.76	.73	.75	.76	.74	-.16	-.33	.33	.43
OTIS	RAW	67	.85	.85	.77	.84	.79	.83	.82	.86	.90	.98	-.29	-.36	.36	.62
GRADES	SPELLING	68	.66	.62	.74	.58	.64	.60	.58	.65	.68	.69	-.35	-.18	.36	.63
	ARITH	69	.55	.50	.58	.56	.61	.50	.49	.59	.60	.62	-.26	-.23	.29	.49
	READING	70	.65	.66	.71	.59	.64	.64	.63	.66	.71	.71	-.32	-.24	.30	.55
	LANG	71	.57	.56	.63	.55	.60	.55	.51	.59	.62	.68	-.36	-.25	.36	.55
	SOCIAL ST	72	.57	.55	.60	.52	.60	.56	.53	.60	.63	.63	-.27	-.25	.31	.50
MEAN			20.4	19.6	28.3	16.8	16.6	28.0	20.9	22.6	59.4	92.4	11.2	13.6	5.38	5.39
SD			8.81	9.35	13.9	8.12	7.24	11.8	9.68	9.87	17.8	14.9	1.77	1.43	2.81	3.23

TABLE 4 (Continued)

51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
.52																					
.48	.56																				
.57	.76	.48																			
.46	.65	.47	.69																		
.47	.63	.36	.68	.76																	
.29	.43	.46	.35	.45	.27																
.21	.34	.29	.34	.49	.47	.43															
.30	.41	.41	.33	.43	.34	.47	.46														
.36	.48	.42	.49	.63	.56	.54	.59	.51													
.16	.16	.26	.17	.18	.14	.23	.08	.26	.16												
.46	.56	.61	.55	.48	.38	.45	.26	.41	.32	.30											
.49	.54	.30	.66	55	.63	.07	.24	.11	.38	.05	.34										
.54	.77	.46	.77	.67	.66	.40	.40	.38	.53	.20	.45	.63									
.55	.81	.48	.78	.63	.69	.39	.39	.38	.49	.22	.51	.56	.78								
.57	.66	.53	.69	.51	.53	.36	.30	.37	.38	.21	.58	.57	.67	.73							
.65	.79	.56	.79	.63	.69	.40	.32	.43	.51	.25	.55	.60	.78	.85	.76						
.42	.69	.38	.60	.57	.61	.33	.39	.44	.46	.12	.39	.36	.65	.73	.48	.68					
.40	.62	.44	.57	.52	.51	.44	.35	.50	.46	.22	.46	.21	.58	.62	.49	.61	.75				
.51	.70	.42	.63	.53	.54	.43	.39	.47	.40	.17	.42	.31	.66	.75	.57	.70	.83	.76			
.45	.62	.43	.57	.52	.49	.44	.32	.48	.47	.20	.43	.19	.59	.66	.47	.66	.79	.82	.84		
.45	.60	.40	.51	.46	.50	.40	.35	.47	.39	.16	.39	.15	.55	.65	.50	.62	.79	.83	.83	.84	
155	164	181	360	348	194	238	514	136	358	901	160		534	205	218	300	595	505	559	516	510
105	593	434	210	987	105	779	149	519	958	108	484		246	104	786	138	303	273	274	282	294

TABLE 5
Partial Correlations of Learning Tasks, Equated by Race

		VAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14
TASK 1	ERRORS	01														
	CURVE	02	.37													
	FIT	03	.09	.15												
TASK 2	ERRORS	04	.05	.04	-.06											
	CURVE	05	.04	.02	-.04	.92										
	FIT	06	.04	.11	-.03	.12	.06									
TASK 3	ERRORS	07	.16	.09	.08	.05	.05	-.04								
	CURVE	08	.17	.08	.08	.04	.05	-.04	.94							
	FIT	09	.13	.15	.03	.08	.11	-.03	.46	.45						
TASK 4	ERRORS	10	.18	.05	.08	.02	.02	-.07	.10	.11	.09					
	CURVE	11	-.04	-.00	-.00	-.04	-.04	-.04	.02	.02	.01	.69				
	FIT	12	.08	.01	.01	.10	.11	-.06	.05	.03	-.04	-.16	-.36			
TASK 5	ERRORS	13	.20	.12	-.05	.00	-.01	.05	.18	.15	.04	-.01	-.00	-.07		
	CURVE	14	.11	.07	-.03	-.01	-.03	-.02	.14	.11	.04	-.01	.02	-.13	.74	
	FIT	15	-.00	-.03	.02	-.04	-.06	.06	.00	.03	.08	.04	.02	.05	-.21	-.20
TASK 6	ERRORS	16	.19	.16	.02	-.00	-.02	.20	.05	.07	.01	.18	.06	.02	.17	.13
	CURVE	17	.05	.11	-.01	-.05	-.06	.16	-.02	-.01	.01	.08	.00	-.02	.02	.03
	FIT	18	.05	-.02	.04	.02	.03	.04	-.04	-.03	.09	.01	-.08	.13	.04	.03
TASK 7	ERRORS	19	.20	.08	.02	.10	.09	.05	.04	.05	.14	.15	.00	-.05	.15	.11
	CURVE	20	.18	.04	-.06	.08	.06	.01	-.03	-.03	.09	.10	-.03	-.06	.15	.13
	FIT	21	-.02	-.02	.14	-.05	-.03	.08	.11	.11	.02	.05	.09	.01	-.02	-.02
TASK 8	ERRORS	22	.17	.14	.02	-.01	-.02	-.01	.15	.15	.06	.16	.09	-.06	.17	.16
	CURVE	23	.14	.11	.07	.07	.07	.01	.12	.12	.08	.16	.08	.00	.15	.13
	FIT	24	.09	.05	.03	.04	.03	.12	.06	.04	.03	.03	.03	-.05	.21	.06
TASK 9	ERRORS	25	.23	.17	-.01	.03	.02	.08	.07	.04	.00	.03	-.05	-.05	.45	.31
	CURVE	26	.02	.14	-.00	.07	.06	.10	.08	.07	.07	.02	-.00	-.16	.26	.26
	FIT	27	.11	-.08	-.07	.08	.07	.03	.05	.06	-.00	-.03	-.11	.30	.10	.13
TASK 10	ERRORS	28	.14	.07	.05	.06	.05	.03	.17	.15	-.00	.07	-.03	.07	.03	.02
	CURVE	29	.06	.01	.05	.03	.02	-.03	.11	.09	-.01	-.02	-.06	-.01	-.00	-.00
	FIT	30	.17	.05	-.04	.02	-.01	-.01	-.05	-.05	.08	.18	.10	-.04	.20	.23
TASK 11	ERRORS	31	.19	.15	.06	-.02	-.03	.04	.03	.01	.12	.13	.07	-.09	.21	.17
	CURVE	32	.06	.09	.04	-.01	-.01	.01	.02	-.00	.06	.01	.00	-.07	.14	.10
	FIT	33	.04	-.02	.29	.01	.02	-.07	-.01	-.02	.14	.12	.10	.04	-.04	-.02
TASK 12	ERRORS	34	.20	.19	.13	.04	.03	.04	.14	.12	.02	.27	.14	-.03	.20	.16
	CURVE	35	.18	.18	.11	-.00	-.01	.07	.16	.14	.05	.23	.15	-.02	.17	.14
	FIT	36	.23	.09	.09	.10	.11	.00	.32	.34	.05	.20	.01	.12	.18	.17

TABLE 5 (Continued)

15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
.03																					
-.00	.76																				
.10	-.06	-.24																			
-.03	.15	.01	.15																		
-.05	.07	-.03	.13	.81																	
.03	.04	.10	-.01	-.03	-.22																
-.04	.10	-.02	.04	.16	.14	.00															
-.07	.11	-.01	.03	.19	.15	.00	.78														
.02	-.02	-.07	.01	-.01	-.01	.07	.35	.28													
-.11	.21	.03	.03	.17	.13	.05	.15	.11	.20												
-.06	.12	.05	.02	.14	.12	.00	.16	.12	.01	.55											
.15	.01	-.01	.11	.06	.07	.05	-.02	.03	-.12	-.15	-.18										
.04	.20	.05	.06	.17	.12	.18	.04	.08	-.08	.12	.10	.08									
.03	.11	.04	-.03	.11	.10	.15	-.02	.02	-.08	.03	.05	-.02	.84								
.05	.20	.14	.03	.10	.06	.03	.05	.04	.08	.21	.11	.06	.02	-.11							
-.06	.16	.06	.19	.32	.22	.05	.32	.28	.14	.27	.18	-.00	.06	-.06	.17						
-.07	.10	.06	.16	.20	.13	.01	.25	.20	.14	.20	.13	-.04	.01	-.07	.05	.84					
-.03	.05	.02	.06	-.01	-.08	.19	-.02	.02	-.04	-.05	-.08	.04	.11	.08	-.07	.09	-.32				
-.05	.14	.04	.01	.19	.15	.11	.30	.29	.11	.20	.10	.04	.17	.10	.10	.39	.22	.21			
-.02	.16	.09	.01	.20	.16	.09	.32	.34	.15	.15	.09	.04	.11	.04	.06	.34	.20	.17	.77		
-.08	.12	.05	-.05	.12	.06	.04	.05	.07	-.01	.17	.16	.05	.06	-.04	.10	.12	.03	.13	.38	.34	

TABLE 6

Partial Correlations of Learning Tasks with Reference Variables, Equated by Race

		VAR	1	2	3	4	5	6	7	8	9	10	11	12	13	14
STANFORD ACH	RDG CMPN	37	-.20	-.16	-.10	-.11	-.10	-.07	-.06	-.04	-.01	-.16	-.08	-.03	-.21	-.17
	VOCAB	38	-.19	-.20	-.08	-.15	-.14	-.07	-.02	-.02	.02	-.12	-.06	-.05	-.14	-.09
	SPELLING	39	-.17	-.27	-.06	-.12	-.11	-.07	.01	.00	.00	-.08	-.06	-.01	-.20	-.11
	ARITH REAS	40	-.20	-.15	-.08	-.09	-.09	-.11	-.05	-.04	-.02	-.16	-.05	-.03	-.16	-.11
	COMPUTN	41	-.22	-.15	-.06	-.13	-.12	-.11	-.07	-.05	-.06	-.17	-.08	-.03	-.20	-.16
	SOCIAL ST	42	-.20	-.15	-.11	-.13	-.13	-.02	-.02	-.03	-.01	-.17	-.06	-.05	-.12	-.08
	SCIENCE	43	-.21	-.19	-.09	-.16	-.15	-.09	.01	-.00	-.02	-.11	-.01	-.07	-.09	-.04
	STUDY SK	44	-.27	-.20	-.11	-.12	-.12	-.07	-.07	-.06	-.04	-.19	-.04	-.09	-.14	-.09
	MD GRADE EQ	45	-.23	-.21	-.11	-.16	-.15	-.10	-.05	-.04	-.02	-.16	-.06	-.06	-.15	-.10
OTIS	IQ	46	-.25	-.23	-.08	-.15	-.13	-.09	-.06	-.06	-.04	-.19	-.06	-.02	-.14	-.08
ROTE MEMORY	TASK 5	47	.14	.12	.04	-.00	-.01	-.05	.06	.05	.04	.03	-.02	.09	.20	-.09
	TASK 11	48	.19	.15	.04	.01	.01	.12	.00	-.01	.11	.20	.11	-.03	.10	.07
	REF B	49	-.09	-.12	-.05	-.05	-.07	.02	-.05	-.07	-.08	-.09	-.09	.03	-.17	-.16
	REF H	50	-.16	-.16	-.05	-.06	-.05	.00	-.03	-.02	-.07	-.08	.02	-.02	-.26	-.19
REASON	REF C	51	-.23	-.19	-.09	-.15	-.15	-.07	-.07	-.10	-.05	-.19	-.05	-.08	-.09	-.06
	PMA RW	52	-.27	-.21	-.03	-.07	-.05	-.09	-.08	-.05	-.06	-.12	-.03	-.01	-.28	-.18
	PMA RF	53	-.20	-.12	-.07	-.10	-.09	-.11	-.25	-.24	-.16	-.22	-.09	-.05	-.14	-.09
ARI REAS	REF E	54	-.25	-.19	-.03	-.06	-.06	-.02	-.07	-.07	-.03	-.18	-.04	-.08	-.13	-.09
NUMBER	PMA N	55	-.19	-.16	-.05	-.03	-.02	-.02	-.15	-.13	-.02	-.19	-.08	.07	-.19	-.10
	PST 1	56	-.17	-.14	-.10	-.03	-.02	.01	-.02	-.02	.05	-.09	-.05	.02	-.20	-.13
PERCEPT	PMA P	57	-.26	-.08	-.04	-.12	-.11	-.13	-.11	-.12	-.05	-.17	.02	-.00	-.20	-.09
	REF G	58	-.14	-.11	.02	.04	.04	.01	-.05	-.05	.03	-.15	-.12	.00	-.15	-.10
	REF D	59	-.23	-.23	-.07	-.14	-.11	-.02	-.06	-.06	.01	-.08	.05	.02	-.19	-.12
	REF I	60	-.21	-.10	-.01	-.02	-.00	-.02	-.16	-.14	.02	-.14	-.06	.01	-.19	-.08
SPACE	REF F	61	-.15	-.08	-.05	-.02	-.03	.02	-.09	-.10	-.03	-.02	.09	.07	-.03	.02
	PMA S	62	-.26	-.14	-.11	-.10	-.10	-.12	-.18	-.18	-.17	-.20	-.01	.07	-.15	-.12
RACE		63	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
WARM UP	REF A	64	-.20	-.15	.02	-.08	-.07	-.04	-.03	-.01	.00	-.12	.00	-.08	-.22	-.12
VERBAL.	PMA VW	65	-.25	-.18	-.06	-.10	-.09	-.03	-.04	-.05	-.01	-.16	-.04	-.08	-.22	-.13
	PMA VP	66	-.26	-.17	-.10	-.09	-.09	-.15	-.04	-.03	-.02	-.17	-.04	-.05	-.04	.01
OTIS	RAW	67	-.26	-.24	-.08	-.15	-.14	-.11	-.06	-.06	-.06	-.19	-.05	-.04	-.14	-.08
GRADES	SPELLING	68	-.20	-.19	-.03	-.08	-.06	-.02	-.04	-.05	-.03	-.12	-.03	-.02	-.23	-.13
	ARITH	69	-.31	-.23	-.05	-.07	-.04	-.07	-.07	-.07	.02	-.20	-.09	-.02	-.26	-.17
	READING	70	-.26	-.18	.01	-.13	-.11	-.04	.00	-.03	.00	-.13	-.01	-.09	-.19	-.08
	LANG	71	-.27	-.21	-.01	-.10	-.07	-.04	-.10	-.11	-.06	-.17	-.03	-.03	-.27	-.15
	SOCIAL ST	72	-.28	-.22	-.04	-.11	-.09	-.09	-.01	-.02	.02	-.20	-.07	-.08	-.24	-.14

TABLE 6 (Continued)

15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
.02	-.26	-.04	-.14	-.23	-.18	-.02	-.18	-.17	-.02	-.25	-.18	-.05	-.25	-.13	-.13	-.30	-.23	-.05	-.26	-.26	-.10
.03	-.25	-.07	-.09	-.17	-.13	-.00	-.10	-.09	.01	-.21	-.15	-.06	-.25	-.09	-.12	-.23	-.16	-.06	-.19	-.19	-.10
-.00	-.27	-.11	-.01	-.09	-.05	.06	-.14	-.14	-.11	-.32	-.21	.03	-.12	.02	-.17	-.24	-.17	-.04	-.23	-.25	-.13
.00	-.21	-.05	-.13	-.20	-.17	-.00	-.19	-.20	-.09	-.21	-.18	-.10	-.22	-.08	-.15	-.29	-.19	-.03	-.28	-.28	-.12
-.03	-.24	-.06	-.13	-.20	-.14	-.05	-.18	-.21	-.05	-.28	-.18	-.07	-.20	-.06	-.19	-.32	-.19	-.10	-.35	-.31	-.17
.04	-.24	-.04	-.19	-.26	-.22	.02	-.12	-.13	.02	-.20	-.11	-.06	-.21	-.04	-.16	-.28	-.19	-.07	-.20	-.21	-.11
-.00	-.27	-.09	-.15	-.22	-.18	-.03	-.10	-.11	.02	-.20	-.12	-.05	-.24	-.09	-.17	-.25	-.16	-.05	-.16	-.16	-.10
-.02	-.29	-.08	-.17	-.24	-.20	-.04	-.21	-.24	.01	-.24	-.14	-.08	-.29	-.14	-.17	-.33	-.24	-.06	-.27	-.27	-.14
-.01	-.29	-.08	-.15	-.21	-.16	-.03	-.17	-.16	-.02	-.26	-.19	-.07	-.27	-.10	-.18	-.31	-.21	-.06	-.26	-.27	-.15
-.02	-.29	-.07	-.16	-.18	-.13	.02	-.19	-.16	-.06	-.28	-.22	-.07	-.24	-.08	-.18	-.29	-.19	-.09	-.29	-.27	-.14
.04	.13	.03	-.03	.05	.01	.02	.04	.04	.15	.33	.09	-.11	.03	.02	.02	.16	.14	.03	.13	.13	.07
.02	.15	.06	.04	.24	.19	.04	.14	.17	.03	.07	.00	.10	.07	-.00	.13	.27	-.08	.25	.25	.24	.09
-.02	-.08	-.01	-.07	-.16	-.06	-.13	-.12	-.12	-.00	-.14	-.13	-.05	-.07	-.01	-.12	-.35	-.22	-.12	-.33	-.26	-.13
-.08	-.21	-.12	-.07	-.19	-.13	.01	-.19	-.17	-.10	-.30	-.19	-.06	-.09	.02	-.14	-.31	-.18	-.09	-.29	-.25	-.11
-.03	-.17	-.03	-.06	-.13	-.12	-.02	-.17	-.18	-.08	-.16	-.11	-.06	-.22	-.09	-.14	-.24	-.15	-.04	-.27	-.24	-.22
.04	-.33	-.09	-.08	-.18	-.13	.09	-.19	-.19	-.11	-.43	-.25	.02	-.22	-.08	-.22	-.29	-.19	-.04	-.23	-.22	-.14
.08	-.23	-.03	-.16	-.25	-.18	-.12	-.36	-.32	-.08	-.25	-.22	-.03	-.24	-.13	-.08	-.31	-.16	-.14	-.30	-.27	-.19
-.09	-.24	-.06	-.05	-.16	-.12	.07	-.19	-.18	-.02	-.26	-.18	-.07	-.19	-.04	-.23	-.28	-.18	.01	-.27	-.25	-.16
-.01	-.14	.00	-.03	-.05	-.03	.03	-.20	-.15	-.09	-.34	-.24	.06	-.13	-.04	-.11	-.25	-.16	.01	-.30	-.23	-.15
.00	-.09	.02	-.09	-.13	-.11	.02	-.13	-.14	-.03	-.25	-.20	-.03	-.11	-.05	-.06	-.25	-.18	-.04	-.27	-.23	-.11
.06	-.22	.02	-.09	-.19	-.16	-.04	-.27	-.22	-.10	-.36	-.15	.02	-.22	-.08	-.14	-.26	-.13	-.06	-.30	-.21	-.23
.02	-.10	.02	.02	-.12	-.12	.00	-.13	-.16	.02	-.20	-.13	-.05	-.10	-.00	.03	-.19	-.08	-.03	-.25	-.21	-.17
.14	-.22	-.08	.04	-.18	-.18	.03	-.19	-.15	-.11	-.29	-.21	-.02	-.12	-.03	-.17	-.20	-.08	-.09	-.36	-.27	-.25
-.03	-.13	.03	.06	-.06	-.05	-.03	-.20	-.20	-.10	-.30	-.15	.06	-.07	-.02	-.01	-.23	-.13	.02	-.34	-.27	-.16
.06	-.05	-.04	-.05	-.12	-.06	-.03	-.19	-.12	-.10	-.13	-.15	.10	-.08	-.07	-.07	-.19	-.14	.02	-.21	-.22	-.09
.07	-.21	-.05	-.16	-.30	-.23	.02	-.30	-.29	-.11	-.21	-.15	.06	-.18	-.08	-.13	-.31	-.16	-.05	-.31	-.28	-.23
.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
-.01	-.24	-.03	-.06	-.14	-.11	.10	-.17	-.16	-.12	-.36	-.20	-.03	-.15	-.02	-.16	-.24	-.14	-.03	-.22	-.17	-.11
-.04	-.34	-.09	-.12	-.23	-.18	.01	-.16	-.16	-.05	-.33	-.20	-.07	-.24	-.04	-.16	-.29	-.20	-.07	-.27	-.25	-.21
.03	-.24	.01	-.14	-.24	-.18	.00	-.12	-.11	.01	-.19	-.11	-.02	-.27	-.11	-.13	-.22	-.10	-.10	-.19	-.16	-.14
-.04	-.29	-.08	-.16	-.20	-.14	.02	-.19	-.17	-.07	-.28	-.21	-.08	-.24	-.08	-.19	-.28	-.17	-.07	-.27	-.25	-.12
-.01	-.31	-.13	-.08	-.17	-.11	.07	-.17	-.17	-.14	-.39	-.27	.07	-.11	.04	-.21	-.37	-.32	-.04	-.23	-.23	-.17
-.00	-.34	-.10	-.08	-.21	-.16	.00	-.28	-.26	-.17	-.39	-.23	.03	-.18	-.00	-.28	-.44	-.32	-.05	-.34	-.32	-.19
-.03	-.34	-.08	-.13	-.22	-.17	.10	-.15	-.14	-.15	-.41	-.23	.03	-.20	-.00	-.27	-.32	-.22	-.07	-.24	-.24	-.21
-.02	-.32	-.08	-.08	-.17	-.11	.04	-.22	-.19	-.17	-.42	-.25	.03	-.19	-.01	-.26	-.39	-.29	-.06	-.33	-.32	-.22
-.00	-.36	-.10	-.11	-.23	-.18	.04	-.20	-.21	-.14	-.38	-.24	.02	-.20	-.01	-.25	-.40	-.28	-.06	-.32	-.30	-.24

TABLE 7
Partial Correlations of Reference Variables, Equated by Race
(N = 240.)

		VAR	37	38	39	40	41	42	43	44	45	46	47	48	49	50
STANFORD	RDG CMPN	37														
ACH	VOCAB	38	.47													
	SPELLING	39	.39	.43												
	ARITH REAS	40	.39	.40	.37											
	COMPUTN	41	.39	.39	.42	.42										
	SOCIAL ST	42	.47	.50	.38	.39	.37									
	SCIENCE	43	.44	.50	.40	.39	.38	.53								
	STUDY SK	44	.46	.44	.38	.42	.45	.49	.45							
	MD GRADE EQ	45	.52	.54	.48	.46	.47	.53	.54	.53						
OTIS	IQ	46	.50	.49	.47	.45	.47	.48	.49	.51	.55					
ROTE	TASK 5	47	.13	-.15	-.24	-.06	-.12	-.10	-.13	-.11	-.13	-.16				
MEMORY	TASK 11	48	.21	-.18	-.16	-.20	-.21	-.24	-.22	-.20	-.25	-.23	.05			
	REF B	49	.19	.17	.16	.17	.22	.14	.14	.19	.19	.17	-.13	-.19		
	REF H	50	.26	.24	.40	.24	.34	.23	.23	.27	.29	.34	-.22	-.19	.34	
REASON	REF C	51	.26	.27	.24	.28	.31	.28	.30	.33	.34	.36	-.10	-.24	.21	.19
	PMA RW	52	.40	.37	.44	.38	.44	.37	.36	.42	.42	.48	-.24	-.19	.12	.35
	PMA RF	53	.28	.26	.19	.29	.38	.28	.27	.35	.33	.38	-.11	-.24	.19	.23
ARI REAS	REF E	54	.36	.31	.33	.36	.38	.34	.33	.39	.38	.39	-.10	-.17	.17	.27
NUMBER	PMA N	55	.25	.21	.30	.26	.34	.18	.16	.25	.25	.29	-.22	-.12	.17	.26
	PST 1	56	.27	.27	.34	.28	.36	.23	.22	.25	.28	.32	-.19	-.13	.19	.25
PERCEPT	PMA P	57	.25	.20	.24	.24	.35	.25	.23	.31	.27	.35	-.20	-.16	.15	.27
	REF G	58	.17	.13	.20	.12	.21	.10	.06	.16	.12	.16	-.16	-.11	.17	.28
	REF D	59	.24	.24	.32	.23	.37	.20	.17	.28	.27	.39	-.15	-.13	.26	.31
	REF I	60	.24	.14	.28	.19	.30	.13	.08	.21	.19	.27	-.27	-.14	.20	.32
SPACE	REF F	61	.16	.18	.09	.20	.19	.17	.16	.18	.19	.21	-.05	-.06	.07	.07
	PMA S	62	.27	.22	.20	.33	.36	.28	.25	.34	.29	.35	-.03	-.22	.19	.21
RACE		63	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
WARM UP	REF A	64	.37	.32	.36	.31	.35	.35	.31	.35	.36	.41	-.22	-.16	.17	.34
VERBAL	PMA VW	65	.47	.47	.48	.40	.43	.49	.47	.46	.50	.52	-.27	-.19	.20	.36
	PMA VP	66	.35	.40	.26	.30	.33	.43	.40	.40	.41	.42	-.05	-.23	.14	.16
OTIS	RAW	67	.49	.48	.45	.44	.45	.48	.47	.49	.53	.64	-.17	-.25	.16	.33
GRADES	SPELLING	68	.44	.40	.55	.35	.43	.39	.38	.43	.45	.49	-.28	-.11	.24	.46
	ARITH	69	.42	.37	.46	.42	.49	.37	.37	.45	.46	.50	-.21	-.19	.22	.38
	READING	70	.46	.47	.54	.39	.47	.47	.45	.47	.52	.54	-.26	-.18	.19	.40
	LANG	71	.45	.44	.53	.43	.49	.43	.40	.47	.50	.57	-.32	-.22	.30	.46
	SOCIAL ST	72	.49	.46	.52	.43	.51	.47	.44	.51	.54	.55	-.24	-.23	.26	.43

TABLE 7 (Continued)

51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
.26																					
.32	.40																				
.25	.41	.28																			
.19	.35	.30	.33																		
.16	.29	.17	.27	.42																	
.26	.39	.44	.31	.41	.23																
.09	.21	.21	.18	.36	.32	.41															
.25	.35	.38	.25	.37	.27	.47	.43														
.17	.28	.30	.24	.42	.32	.51	.50	.47													
.14	.13	.24	.14	.15	.11	.22	.07	.25	.14												
.29	.38	.51	.32	.29	.16	.43	.18	.37	.19	.29											
.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00										
.23	.43	.27	.35	.32	.26	.36	.24	.31	.30	.17	.23	.00									
.28	.50	.31	.41	.32	.33	.35	.25	.32	.27	.19	.32	.00	.43								
.29	.35	.35	.32	.20	.17	.32	.16	.31	.16	.18	.39	.00	.31	.41							
.35	.46	.37	.40	.30	.31	.35	.18	.37	.28	.22	.34	.00	.40	.51	.42						
.24	.49	.27	.36	.37	.38	.30	.30	.40	.33	.11	.26	.00	.43	.53	.27	.46					
.29	.50	.37	.43	.40	.38	.42	.30	.48	.38	.21	.38	.00	.44	.49	.37	.48	.68				
.35	.54	.33	.43	.36	.34	.41	.31	.43	.29	.15	.31	.00	.47	.58	.39	.52	.72	.70			
.36	.51	.37	.44	.42	.37	.42	.27	.45	.40	.19	.37	.00	.47	.55	.36	.54	.72	.78	.78		
.37	.52	.36	.41	.38	.40	.39	.31	.46	.34	.15	.34	.00	.46	.56	.41	.53	.73	.80	.78	.81	

APPENDIX D

FACTOR ANALYSIS MATRICES

TABLE 8
Transformation Matrix

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
I	.22	.35	.27	.00	-.53	.33	-.12	.23	-.63	.27	.09	-.22	.06	-.54
II	-.34	.45	.14	.47	-.29	.37	-.40	-.32	-.00	-.18	.00	.09	-.32	.10
III	-.12	-.14	.41	.11	-.23	-.21	-.13	.11	-.22	.27	-.26	.46	-.07	.06
IV	.16	-.30	-.00	.66	.14	-.28	.19	-.37	-.41	-.31	-.17	.15	.29	-.21
V	-.25	-.01	-.17	-.01	.11	-.17	.46	-.02	-.09	-.02	-.28	-.18	-.32	-.27
VI	-.14	-.54	-.33	.21	.22	.49	-.32	.01	-.04	.57	.28	-.01	-.04	.05
VII	-.20	-.05	-.13	-.07	-.55	.08	.49	-.04	.12	-.13	.41	.25	.15	.13
VIII	.28	-.01	-.20	.06	-.28	-.03	-.19	-.12	.31	-.05	-.15	-.14	-.09	-.16
IX	.62	.01	.02	.07	.00	.05	.07	.01	.18	.05	.03	.01	.07	.01
X	.27	.30	-.52	.01	.11	-.06	-.25	-.02	-.01	.32	.29	.60	-.25	.12
XI	-.07	-.13	.25	.04	.14	-.33	-.10	-.08	.23	-.03	.59	-.08	-.24	-.18
XII	-.15	-.06	-.00	-.17	.26	.16	-.03	.11	.29	-.15	-.23	.40	.20	-.60
XIII	-.02	.18	.07	-.46	-.00	.11	.01	-.75	.18	.25	.16	-.14	-.21	-.24
XIV	-.28	.31	-.43	.08	-.04	-.40	-.27	.27	-.14	.40	.04	-.13	.65	-.19

TABLE 9
Final Rotated Factor Matrix
(Variables 1–36, 47, and 48 were reflected.)

		VAR	I	II	III	IV	V
TASK 1	ERRORS	1	.40	-.02	.04	.06	.06
	CURVE	2	.25	.02	-.06	-.01	-.01
	FIT	3	.16	.09	.03	-.02	-.08
TASK 2	ERRORS	4	.26	-.08	.00	-.06	.10
	CURVE	5	.20	-.07	.02	-.07	.08
	FIT	6	.12	-.06	.06	-.09	.06
TASK 3	ERRORS	7	.12	.00	.16	.02	.00
	CURVE	8	.10	-.03	.16	.05	.02
	FIT	9	-.07	-.07	.13	-.04	-.09
TASK 4	ERRORS	10	.28	.04	.03	-.02	-.01
	CURVE	11	.09	.07	-.05	-.02	-.05
	FIT	12	.13	-.10	-.14	.07	.05
TASK 5	ERRORS	13	.42	.04	-.16	.10	-.00
	CURVE	14	.35	.03	-.13	.04	-.02
	FIT	15	.00	.11	-.08	-.01	-.08
TASK 6	ERRORS	16	.33	-.16	-.09	.19	.06
	CURVE	17	-.02	-.11	-.05	.07	-.03
	FIT	18	.21	.09	.21	-.06	-.19
TASK 7	ERRORS	19	.23	-.13	.19	.12	-.02
	CURVE	20	.20	-.14	.10	.08	.09
	FIT	21	.04	-.10	.02	.04	.04
TASK 8	ERRORS	22	.24	-.01	.18	.01	-.01
	CURVE	23	.22	-.02	.09	-.04	.02
	FIT	24	.06	-.03	.06	-.06	-.08
TASK 9	ERRORS	25	.49	.13	-.05	.02	-.00
	CURVE	26	.39	.16	.11	-.00	-.08
	FIT	27	.08	-.05	-.22	.09	.10
TASK 10	ERRORS	28	.21	-.03	.00	.11	.05
	CURVE	29	-.03	.04	.06	-.04	.02
	FIT	30	.23	.00	-.03	-.15	-.11
TASK 11	ERRORS	31	.24	.06	.16	-.05	-.16
	CURVE	32	.08	.06	.08	.00	-.19
	FIT	33	.07	-.03	.03	.02	-.07
TASK 12	ERRORS	34	.32	.07	.06	-.00	.05
	CURVE	35	.26	.01	.04	-.00	.01
	FIT	36	.17	-.10	.03	.08	.08

TABLE 9 (Continued)

VI	VII	VIII	IX	X	XI	XII	XIII	XIV	h^2
-.08	.01	-.00	.03	.07	-.07	.03	.09	.21	.40
.07	-.01	.10	.17	.03	.16	.03	.01	.25	.25
.02	.01	-.00	.07	-.11	.02	-.31	.18	.06	.27
.05	-.02	-.11	.26	-.01	-.02	.06	-.17	.04	.19
.09	-.05	-.09	.26	.01	.00	.02	-.20	.04	.17
-.10	.02	-.16	.11	.05	.01	.00	.03	-.05	.12
-.04	-.01	-.14	.02	-.04	-.02	.02	-.00	.39	.31
-.05	.03	-.19	.01	-.08	-.04	.03	-.01	.36	.29
.11	.00	-.30	.00	.08	.05	.00	-.03	.28	.35
-.05	-.01	.19	.04	.03	.08	-.08	.21	.05	.23
.02	-.00	.26	.06	-.05	.22	-.07	.16	-.02	.22
-.08	.04	.20	-.04	.04	-.44	-.06	.01	.10	.37
.18	-.10	.15	-.01	.21	.21	.19	-.06	.47	.62
.19	-.08	-.02	.01	.17	.22	.17	.03	.23	.35
.08	.02	-.22	-.12	.07	-.25	-.03	.03	-.23	.25
-.04	.06	-.25	-.04	.19	.16	.00	.11	.02	.52
.06	.07	-.20	-.02	.06	.18	-.02	.05	-.04	.13
.07	.02	.11	.05	-.08	-.31	-.04	-.02	-.04	.28
.11	-.01	-.08	.02	-.01	-.04	.15	.08	.05	.30
-.00	-.04	.06	.12	-.02	-.05	.29	.02	.06	.28
.08	.05	-.12	.06	-.16	-.05	-.17	.17	.07	.18
.01	-.02	.10	-.07	.08	.03	.15	.28	.05	.39
-.02	.01	.13	-.04	.10	.00	.10	.36	.00	.36
-.02	.11	.49	-.09	.04	.02	.06	.05	.24	.36
.08	-.07	.14	-.06	.31	.28	.05	-.13	.27	.74
.11	-.01	-.33	.01	.10	.31	.08	-.10	-.01	.51
.02	-.10	.03	.09	.01	-.46	.13	.09	.21	.46
-.07	-.05	-.24	.11	.05	-.02	-.11	.10	.05	.33
-.03	-.08	-.24	.15	.01	.08	-.12	-.01	.05	.16
.09	.08	.06	.02	.24	.09	-.03	-.04	.08	.24
.25	.08	.33	-.02	.03	-.02	.15	.24	-.04	.60
.12	.10	.45	-.04	-.03	-.03	.26	.23	-.05	.52
.15	.06	.08	-.09	-.04	-.05	-.51	.14	-.01	.48
.18	-.01	.16	.01	.00	.01	-.04	.35	.04	.60
.09	.07	.07	.05	-.03	.02	.02	.38	.06	.48
.01	.02	.42	.04	-.03	-.05	-.06	-.03	.52	.59

TABLE 9.
Final Rotated Factor Matrix (Continued)
(Variables 1–36, 47, and 48 were reflected.)

		VAR	I	II	III	IV	V
STANFORD ACH	RDG CMPN	37	.60	-.01	-.11	.15	-.01
	VOCAB	38	.61	-.03	-.02	.23	-.03
	SPELLING	39	.54	.09	-.18	.08	.00
	ARITH REAS	40	.66	.10	.03	-.01	-.09
	COMPUTN	41	.56	.14	.07	-.06	-.01
	SOCIAL ST	42	.58	-.06	-.01	.16	-.06
	SCIENCE	43	.58	-.05	-.03	.12	-.08
	STUDY SK	44	.61	-.03	-.02	-.01	.01
	MD GRADE EQ	45	.61	-.02	-.06	.02	-.03
OTIS	IQ	46	.55	.00	.02	.04	.02
ROTE MEMORY	TASK 5	47	.20	.09	-.14	.24	.01
	TASK 11	48	.18	.00	.06	-.09	-.05
	REF B	49	.33	.03	.11	.00	-.06
	REF H	50	.48	.00	-.11	.01	.06
REASON	REF C	51	.49	-.04	.13	-.07	-.00
	PMA RW	52	.54	.07	.02	.09	-.02
	PMA RF	53	.31	-.08	.45	.03	.07
ARI REAS	REF E	54	.66	.15	.02	.01	-.06
NUMBER	PMA N	55	.55	.42	.09	-.04	.04
	PST 1	56	.63	.39	-.05	.03	-.02
PERCEPT	PMA P	57	.07	-.05	.12	-.02	.38
	REF G	58	.24	.02	-.08	.17	.54
	REF D	59	.11	-.07	.23	-.07	.33
	REF I	60	.38	.08	-.08	-.07	.44
SPACE	REF F	61	.05	-.05	.34	.11	.00
	PMA S	62	.35	-.03	.46	-.05	-.02
RACE		63	1.00	.00	.00	.00	.00
WARM UP	REF A	64	.63	.07	-.03	.04	.05
VERBAL	PMA VW	65	.56	.03	-.05	.34	-.02
	PMA VP	66	.57	-.11	.16	.16	.05
OTIS	RAW	67	.60	.01	.00	.01	.05
GRADES	SPELLING	68	.36	.05	-.07	.05	-.04
	ARITH	69	.21	-.01	.08	-.03	-.01
	READING	70	.31	-.06	-.03	.08	.03
	LANG	71	.19	-.01	.07	-.04	-.05
	SOCIAL ST	72	.15	-.02	.03	-.00	-.02

TABLE 9. (Continued)

VI	VII	VIII	IX	X	XI	XII	XIII	XIV	h^2
.01	.02	-.03	.19	.06	.06	.03	.14	.02	.82
.05	.01	-.03	.32	-.13	.04	.01	-.05	.04	.94
.11	.09	.01	.17	.07	.16	.01	-.02	-.03	.78
.02	.00	-.01	.20	.06	.00	.02	.10	-.04	.82
.08	.05	-.01	.17	.05	-.03	-.05	.04	-.03	.78
-.00	-.01	.06	.23	.03	.02	-.02	.06	-.02	.84
.04	-.02	.01	.28	.04	.04	-.05	.01	-.04	.85
-.01	.06	-.05	.25	.09	-.02	-.01	.15	-.02	.89
.03	.06	.01	.38	.00	.04	-.02	.06	.00	.99
.00	.06	-.01	.25	.04	-.01	-.02	.04	-.05	.86
.10	-.02	.23	-.19	.07	.02	-.04	-.02	.18	.37
.18	-.05	.29	.13	.05	-.03	-.10	.12	.07	.31
.55	.01	-.03	.02	-.05	-.03	.00	-.03	.01	.56
.36	.05	.01	-.02	.18	.00	.02	.02	.01	.67
.07	.08	-.05	.18	.00	-.05	.01	.05	.02	.51
-.00	-.03	-.05	.00	.27	.15	.01	-.03	.03	.82
-.02	.02	-.21	.00	-.00	-.10	-.07	.13	-.02	.78
.02	-.04	.03	.02	.21	-.02	.05	.06	-.06	.81
-.00	-.05	.04	-.05	.05	.03	-.01	-.04	.01	.83
.01	.05	-.04	.06	-.11	-.03	.02	.05	-.00	.83
-.08	-.07	.25	-.03	.19	-.08	-.11	.06	.02	.64
-.07	.05	-.24	-.07	-.09	-.13	.08	.10	-.10	.79
.09	.14	-.07	.11	-.07	.02	-.06	-.14	.01	.56
.04	-.03	.24	.06	.08	-.03	.01	.01	.11	.74
-.04	-.06	.24	.03	-.13	-.02	.01	.02	.04	.32
.06	-.04	.01	.03	.09	.02	-.04	-.00	.04	.65
.00	.00	.00	.00	.00	.00	.00	.00	.00	100
.09	-.05	.10	.00	.25	.02	.02	-.07	-.03	.81
.03	-.02	-.05	-.05	.12	-.02	.00	.07	-.04	.94
-.04	-.03	.06	.12	.01	-.05	-.09	.00	-.03	.73
.01	.02	-.02	.29	.07	-.02	-.02	.02	-.03	.92
.08	.37	-.04	.02	.00	.08	.03	-.04	-.03	.82
-.12	.47	.07	-.04	-.01	.00	.00	.14	.03	.84
-.06	.37	-.00	.02	.04	-.02	.00	-.03	-.08	.87
.08	.43	.10	.05	.00	.03	-.01	-.10	.08	.90
-.01	.47	-.01	.09	-.07	-.01	.00	.00	-.02	.85

TABLE 9 (Continued)

Interpretation of Factors

I	Race	VIII	Uninterpreted
II	Number	IX	Achievement Scores
III	Nonverbal Reasoning	X	Verbal Reasoning
IV	Vocabulary	XI	Concentration
V	Perceptual Speed	XII	Memory-Task Learning
VI	Rote Memory	XIII	Numerical-Task Learning
VII	Course Marks	XIV	Memory-Task Learning

TABLE 10
Intercorrelations of Factors

		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV
I	Race	1.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
II	Number	.00	1.00	.53	.64	.61	.34	.69	.16	.53	.57	-.04	-.03	.25	.11
III	Nonverbal Reasoning	.00	.53	1.00	.55	.46	.23	.54	.08	.55	.64	-.02	.02	.37	.13
IV	Vocabulary	.00	.64	.55	1.00	.36	.29	.75	.24	.77	.74	-.10	-.16	.13	-.07
V	Perceptual Speed	.00	.61	.46	.36	1.00	.39	.44	.11	.18	.32	.14	-.01	.22	.11
VI	Rote Memory	.00	.34	.23	.29	.39	1.00	.39	.10	.21	.21	-.02	.03	.30	.08
VII	Course Marks	.00	.69	.54	.75	.44	.39	1.00	.19	.63	.77	-.09	-.03	.17	.03
VIII	Uninterpreted	.00	.16	.08	.24	.11	.10	.19	1.00	.22	.04	.13	-.07	-.33	-.18
IX	Achievement Scores	.00	.53	.55	.77	.18	.21	.63	.22	1.00	.72	-.24	-.12	.23	-.07
X	Verbal Reasoning	.00	.57	.64	.74	.32	.21	.77	.04	.72	1.00	-.29	-.07	.22	.07
XI	Concentration	.00	-.04	-.02	-.10	.14	-.02	-.09	.13	-.24	-.29	1.00	.04	.01	-.13
XII	Memory-Task Learning	.00	-.03	.02	-.16	-.01	.03	-.03	-.07	-.12	-.07	.04	1.00	.04	-.07
XIII	Numerical-Task Learning	.00	.25	.37	.13	.22	.30	.17	-.33	.23	.22	.01	.04	1.00	.27
XIV	Memory-Task Learning	.00	.11	.13	-.07	.11	.08	.03	-.18	-.07	.07	-.13	-.07	.27	1.00

TABLE 11

Distribution of Residual Correlations After Extraction of Fourteen Factors

(Coefficients between experimentally dependent variables have been omitted)

(N = 2520)

Res.	Freq.	Res.	Freq.	Res.	Freq.	Res.	Freq.
-.39	0	-.19	0	+.00	407	+.20	3
-.38	0	-.18	1	+.01	246	+.21	0
-.37	0	-.17	3	+.02	185	+.22	0
-.36	1	-.16	2	+.03	124	+.23	0
-.35	0	-.15	2	+.04	74	+.24	0
-.34	0	-.14	5	+.05	56	+.25	1
-.33	0	-.13	8	+.06	35	+.26	0
-.32	0	-.12	5	+.07	36	+.27	0
-.31	0	-.11	5	+.08	34	+.28	0
-.30	0	-.10	14	+.09	7	+.29	0
-.29	1	-.09	22	+.10	15	+.30	0
-.28	0	-.08	13	+.11	11	+.31	0
-.27	0	-.07	26	+.12	10	+.32	0
-.26	0	-.06	37	+.13	3	+.33	0
-.25	0	-.05	59	+.14	6	+.34	0
-.24	0	-.04	82	+.15	4	+.35	0
-.23	0	-.03	136	+.16	1	+.36	0
-.22	0	-.02	163	+.17	0	+.37	0
-.21	1	-.01	283	+.18	0	+.38	0
-.20	1	-.00	390	+.19	2	+.39	0

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