THE SCIENCE OF CAUSE AND EFFECT

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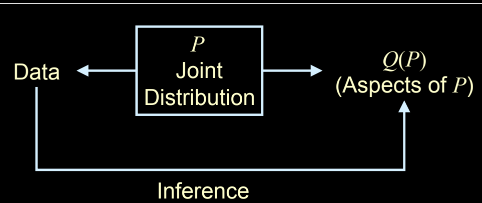
OUTLINE

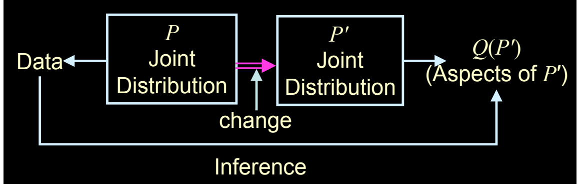
1. The causal revolution – from associations to intervention to counterfactuals
2. The two fundamental laws of causal inference
3. From counterfactuals to problem solving
    a) policy evaluation (ATE, ETT, …)
    b) Mediation
    c) transportability – external validity
    d) missing data
    e) [attribution, selection bias, heterogeneity]

TRADITIONAL STATISTICAL INFERENCE PARADIGM



e.g.,
Infer whether customers who bought product A would also buy product B.
$Q = P(B \mid A)$

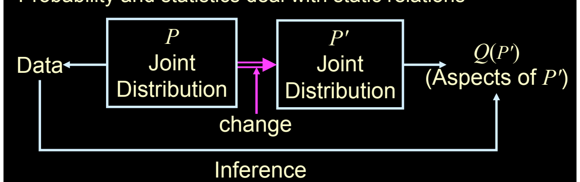
FROM ASSOCIATION TO INTERVENTION



e.g., Estimate $P'(sales)$ if we double the price.
How does P change to P'? New oracle
e.g., Estimate $P'(cancer)$ if we ban smoking.

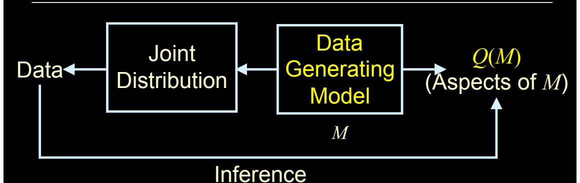
FROM ASSOCIATION TO COUNTERFACTUALS:

Probability and statistics deal with static relations



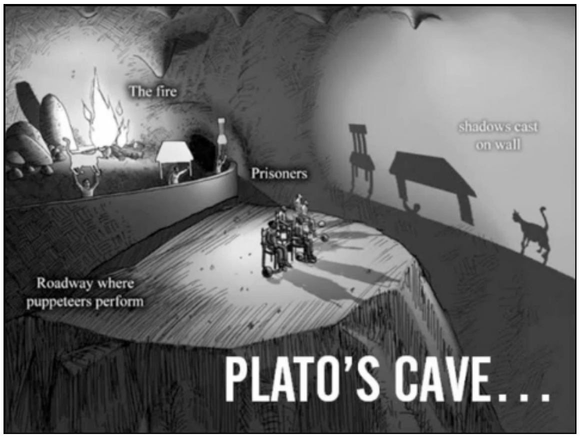
What happens when P changes?
e.g., Estimate the probability that a customer who bought A would buy A if we were to double the price.

THE STRUCTURAL MODEL PARADIGM



M – Invariant strategy (mechanism, recipe, law, protocol) by which Nature assigns values to variables in the analysis.

"A painful de-crowning of a beloved oracle!"



FROM STATISTICAL TO CAUSAL ANALYSIS: THE SHARP BOUNDARY

1. Causal and associational concepts do not mix.

CAUSAL	ASSOCIATIONAL
Spurious correlation	Regression
Randomization / Intervention	Association / Independence
Confounding / Effect	"Controlling for" / Conditioning
Instrumental variable	Odds and risk ratios
Ignorability / Exogeneity	Collapsibility / Granger causality
Explanatory variables	Propensity score

2.

3.

4.

FROM STATISTICAL TO CAUSAL ANALYSIS: 3. THE MENTAL BARRIERS

1. Causal and associational concepts do not mix.

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Spurious correlation	Regression
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Explanatory variables	Propensity score

2. No causes in – no causes out (Cartwright, 1989)

$$\left.\begin{array}{r}\text{data}\\ \text{causal assumptions}\end{array}\right\} \Rightarrow \text{causal conclusions}$$

3. Causal assumptions cannot be expressed in the mathematical language of standard statistics.
4. Non-standard mathematics:
   a) Structural equation models (Wright, 1920; Simon, 1960)
   b) Counterfactuals (Neyman-Rubin (Y_x), Lewis ($x \;\square\!\!\rightarrow Y$))

THE NEW ORACLE: STRUCTURAL CAUSAL MODELS: THE WORLD AS A COLLECTION OF SPRINGS

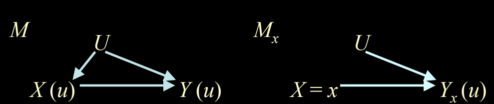
Definition: A structural causal model is a 4-tuple $<V, U, F, P(u)>$, where

- $V = \{V_1, ..., V_n\}$ are endogenous variables
- $U = \{U_1, ..., U_m\}$ are background variables
- $F = \{f_1, ..., f_n\}$ are functions determining V,
  $v_i = f_i(v, u)$ e.g., $y = \alpha + \beta x + u_Y$ Not regression!!!!
- $P(u)$ is a distribution over U

$P(u)$ and F induce a distribution $P(v)$ over observable variables

COUNTERFACTUALS ARE EMBARRASSINGLY SIMPLE

Definition:
$Y_x(u)$: What Y would be had X been x.
$Y_x(u)$ = the solution for Y in a mutilated model M_x, in which the equation for X is replaced by $X = x$.

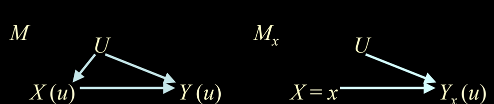


The Fundamental Equation of Counterfactuals:

$$Y_x(u) \triangleq Y_{M_x}(u)$$

EFFECTS OF INTERVENTIONS ARE EMBARRASSINGLY SIMPLE

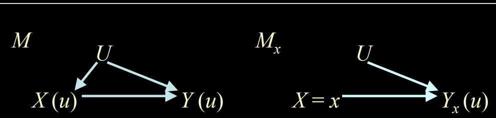
Definition:
The effect of setting X to x, $P(Y = y \mid do(X{=}x))$, is equal to the probability of $Y = y$ in a mutilated model M_x, in which the equation for X is replaced by $X = x$.



The Fundamental Equation of Interventions:

$$P(Y = y \mid do(X = x)) \triangleq P_{M_x}(Y = y) = P(Y_x = y)$$

ESTIMATING THE EFFECTS OF INTERVENTIONS WITHOUT EQUATIONS



The Fundamental Equation of Interventions:

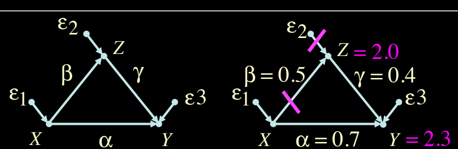
$$P(Y = y \mid do(X = x)) \triangleq P_{M_x}(Y = y)$$

$$P(x,y,u) = P(u)\cancel{P(x \mid u)}P(y \mid x,u)$$

$$P(y,u \mid do(x)) = P(u)P(y \mid x,u)$$ Truncated product

$$P(y \mid do(x)) = \sum_u P(y \mid x,u)P(u)$$ Adjustment formula

READING COUNTERFACTUALS FROM SEM



$X =$ Treatment	$x = \varepsilon_1$
$Z =$ Study Time	~~$z = \beta x + \varepsilon_2$~~ $z = 2.0$
$Y =$ Score	$y = \alpha x + \gamma z + \varepsilon_3$

Data shows: α = 0.7, β = 0.5, γ = 0.4

A student named Joe, measured X = 0.5, Z = 1.0, Y = 1.9

Q_1: What would Joe's score be had he doubled his study time?

Answer: $Y_{Z=2}$ = 0.7 0.5 + 0.4 2.0 + ε_3 = 2.30

THE TWO FUNDAMENTAL LAWS OF CAUSAL INFERENCE

1. The Law of Counterfactuals (and Interventions)

   $$Y_x(u) = Y_{M_x}(u)$$

   (M generates and evaluates all counterfactuals.)

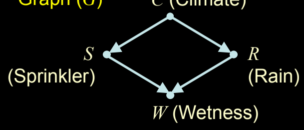
2. The Law of Conditional Independence (d-separation)

   $$(X \text{ sep } Y \mid Z)_{G(M)} \Rightarrow (X \perp\!\!\!\perp Y \mid Z)_{P(v)}$$

   (Separation in the model ⇒ independence in the distribution.)

THE LAW OF CONDITIONAL INDEPENDENCE

Graph (G)



Model (M)

$C = f_C(U_C)$
$S = f_S(C, U_S)$
$R = f_R(C, U_R)$
$W = f_W(S, R, U_W)$

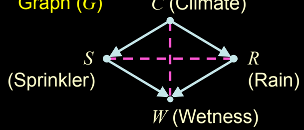
Gift of the Gods

If the U's are independent, the observed distribution $P(C,R,S,W)$ satisfies constraints that are:

(1) independent of the f's and of $P(U)$,
(2) readable from the graph.

D-SEPARATION: NATURE'S LANGUAGE FOR COMMUNICATING ITS STRUCTURE

Graph (G)



Model (M)

$C = f_C(U_C)$
$S = f_S(C, U_S)$
$R = f_R(C, U_R)$
$W = f_W(S, R, U_W)$

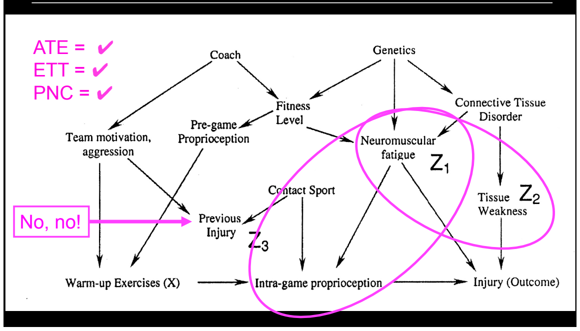
Every missing arrow advertises an independency, conditional on a separating set.

e.g., $C \perp\!\!\!\perp W \mid (S, R)$ $\quad S \perp\!\!\!\perp R \mid C$

Applications:

1. Model testing
2. Structure learning
3. Reducing "what if I do" questions to symbolic calculus
4. Answering scientific questions from the graph

WHAT IF VARIABLES ARE UNOBSERVED? EFFECT OF WARM-UP ON INJURY (Shrier & Platt, 2008)



MATHEMATICAL RESULT #1: (Intervention is a solved problem)

- The estimability of any expression of the form
  $Q = P(y_1, y_2, y_3, \ldots, y_m \mid do(x_1, x_2, \ldots, x_n), Z_1, Z_2, \ldots, Z_k)$
  Can be determined in polynomial time, given any causal graph G with both measured and unmeasured variables.
- If Q is estimable, then its estimand can be derived in polynomial time
- The algorithm is complete

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MEDIATION: A COUNTERFACTUAL TRIUMPH

1. Why decompose effects?
2. What is the definition of direct and indirect effects?
3. What are the policy implications of direct and indirect effects?
4. When can direct and indirect effect be estimated consistently from experimental and nonexperimental data?

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WHY DECOMPOSE EFFECTS?

1. To understand how Nature works
2. To comply with legal requirements
3. To predict the effects of new type of interventions: Signal re-routing and mechanism deactivating, rather than variable fixing

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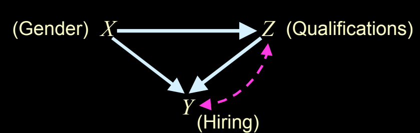
LEGAL IMPLICATIONS OF DIRECT EFFECT

Can data prove an employer guilty of hiring discrimination?



What is the direct effect of X on Y? (CDE)

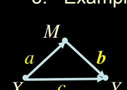
$$E(Y|do(x_1),do(z)) - E(Y|do(x_0),do(z))$$

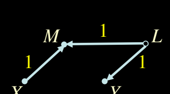
(z-dependent) Adjust for Z? No! No!

Identification is completely solved (Tian & Shpiser, 2006) 23

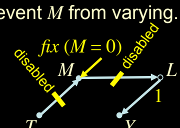
TRADITIONAL MEDIATION ANALYSIS A CURABLE BAD HABIT

1. To prevent M from varying, control for M, the resulting partial regression would be the direct effect.
2. Wrong! "Controlling" does not prevent M from varying.
3. Example:





$M = X + L$, $Y = L$
controlling for $M = 0$
yields $Y = -X$

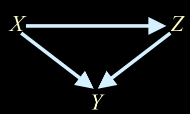


Fixing $M = 0$
yields $Y = L$
independent of X

"The best way to discuss moderation or mediation is to set aside the entire literature on these topics and start from scratch." (Rod McDonald, 2001)

NATURAL INTERPRETATION OF AVERAGE DIRECT EFFECTS

Robins and Greenland (1992), Pearl (2001)



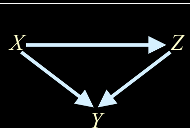
$z = f(x, u)$
$y = g(x, z, u)$

Natural Direct Effect of X on Y: $DE(x_0, x_1; Y)$
The expected change in Y, when we change X from x_0 to x_1 and, for each u, we keep Z constant at whatever value it attained before the change.

$$E[Y_{x_1 Z_{x_0}} - Y_{x_0}]$$

In linear models, DE = Controlled Direct Effect = $\beta(x_1 - x_0)$

DEFINITION OF INDIRECT EFFECTS



$z = f(x, u)$
$y = g(x, z, u)$

No controlled indirect effect

Indirect Effect of X on Y: $IE(x_0, x_1; Y)$
The expected change in Y when we keep X constant, say at x_0, and let Z change to whatever value it would have attained had X changed to x_1.

$$E[Y_{x_0 Z_{x_1}} - Y_{x_0}]$$

In linear models, $IE = TE - DE$

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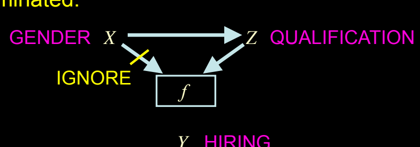
POLICY IMPLICATIONS OF INDIRECT EFFECTS

What is the indirect effect of X on Y?

The effect of Gender on Hiring if sex discrimination is eliminated.



Deactivating a link – a new type of intervention

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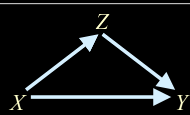
THE MEDIATION FORMULAS IN UNCONFOUNDED MODELS



$z = f(x, u_1)$
$y = g(x, z, u_2)$
u_1 independent of u_2

$$DE = \sum_z [E(Y \mid x_1, z) - E(Y \mid x_0, z)] P(z \mid x_0)$$

$$IE = \sum_z [E(Y \mid x_0, z)[P(z \mid x_1) - P(z \mid x_0)]$$

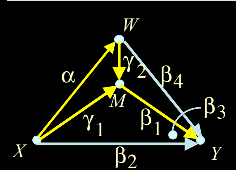
$$TE = E(Y \mid x_1) - E(Y \mid x_0) \qquad TE \neq DE + IE$$

$IE =$ Fraction of responses explained by mediation (sufficient)

Complete identification conditions for confounded models with multiple mediators (Pearl 2001; Shpitser 2013).

$TE - DE =$ Fraction of responses owed to mediation (necessary)

WHAT CAN MEDIATION FORMULA DO FOR PARAMETRIC ANALYSTS?



Multi-mediators non-linear models

$$y = \beta_1 m + \beta_2 x + \beta_3 xm + \beta_4 w + u_1$$
$$m = \gamma_1 x + \gamma_2 w + u_2$$
$$w = \alpha x + u_3$$

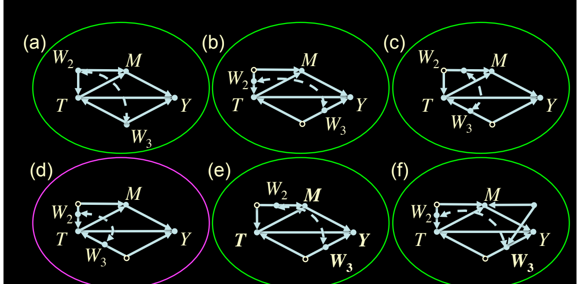
What combination of parameters gives the effect mediated by M?

$$IE(M) = \beta_1(\gamma_1 + \alpha\gamma_2)$$

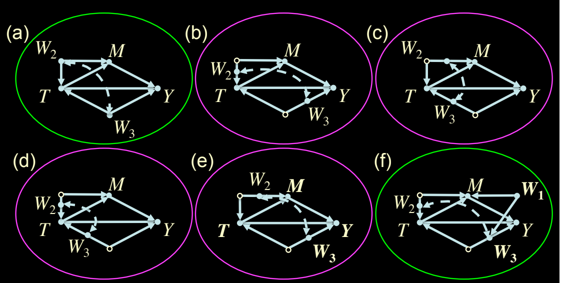
What combination of parameters gives the effect owed to M?

$$TE - DE(M) = (\beta_1 + \beta_3)(\gamma_1 + \alpha\gamma_2)$$

WHEN CAN WE IDENTIFY MEDIATED EFFECTS?



WHEN CAN WE IDENTIFY MEDIATED EFFECTS?



MATHEMATICAL RESULT #2: (Natural mediation is a solved problem)

- Ignorability is not required for identifying natural effects
- The nonparametric estimability of natural (and controlled) direct and indirect effects can be determined mechanically given any causal graph G with both measured and unmeasured variables.
- If NDE (or NIE) is estimable, then its estimand can be derived mechanically in polynomial time.
- The algorithm is complete and was extended to any path-specific effect by Shpitser (2013).

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TRANSPORTABILITY OF KNOWLEDGE ACROSS DOMAINS (with E. Bareinboim)

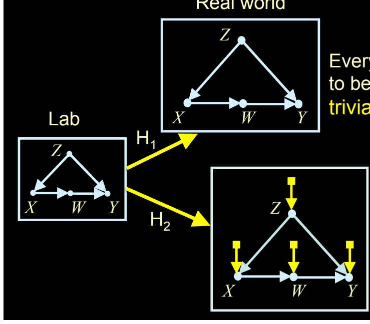
A Theory of Causal Transportability

When can causal relations learned from experiments be transferred to another environment, different from the first, in which no experiment can be conducted.

External Validity – Decades of Literature

- Cox (1958)
- Campbell and Stanley (1963)
- Manski (2007)

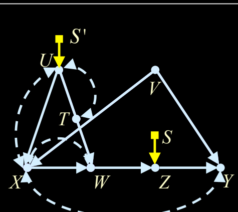
MOVING FROM THE LAB TO THE REAL WORLD . . .



Everything is assumed to be the same, trivially transportable!

Everything is assumed to be the different, not transportable!

RESULT: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE



INPUT: Annotated Causal Graph

S → Factors creating differences

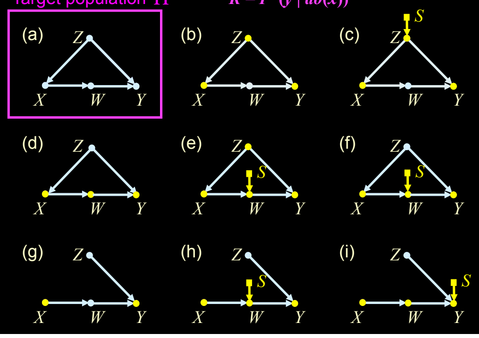
OUTPUT:

1. Transportable or not?
2. Measurements to be taken in the experimental study
3. Measurements to be taken in the target population
4. A transport formula
5. Completeness (Bareinboim, 2012)

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) \sum_w P^*(z \mid w) \sum_t P(w \mid do(w), t) P^*(t)$$

META-SYNTHESIS AT WORK

Target population Π^* $R = P^*(y \mid do(x))$



MATHEMATICAL RESULT #3:
(Transportability and meta-transportability are solved)

- Nonparametric transportability of experimental results from multiple environments can be decided in polynomial time, provided commonalities and differences are encoded in selection diagrams.
- When transportability is feasible, the transport formula can be derived in polynomial time, which specifies the information needed to be extracted from each environment to synthesize a consistent estimate for the target environment.
- The algorithm is complete.

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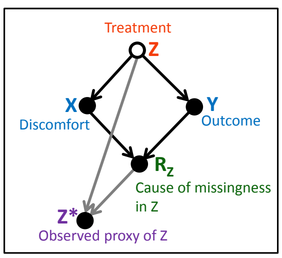
MISSING DATA: FROM A CAUSAL INFERENCE PERSPECTIVE
(Mohan, Pearl & Tian 2013)

- Pervasive in every experimental science.
- Huge literature, powerful software industry, deeply entrenched culture.
- Current practices are based on statistical characterization (Rubin, 1976) of a problem that is inherently causal.
- Needed: (1) theoretical guidance, (2) performance guarantees, and (3) tests of assumptions.

GRAPHICAL MODELS FOR MISSING DATA

X	Y	Z*	R_Z	$P(Z^*,X,Y,R_Z)$
0	0	0	0	0.01
0	0	1	0	0.21
0	1	0	0	0.01
0	1	1	0	0.04
1	0	0	0	0.02
1	0	1	0	0.20
1	1	0	0	0.05
1	1	1	0	0.08
0	0	m	1	0.01
0	1	m	1	0.02
1	0	m	1	0.30
1	1	m	1	0.05

Distribution with missing values



Graph depicting the missingness process

WHAT CAN CAUSAL THEORY DO FOR MISSING DATA?

Q-1. What should the world be like, for a given statistical procedure to produce the expected result?

Q-2. Can we tell from the postulated world whether any method can produce a bias-free result? How?

Q-3. Can we tell from data if the world does not work as postulated?

- To answer these questions, we need models of the world, i.e., process models.
- Statistical characterization of the problem is too crude, e.g., MCAR, MAR, MNAR → testable / untestable; recoverable / non-recoverable

RECOVERABILITY AND TESTABILITY

Recoverability
Given a missingness model G and data D, when is a quantity Q estimable from D without bias?

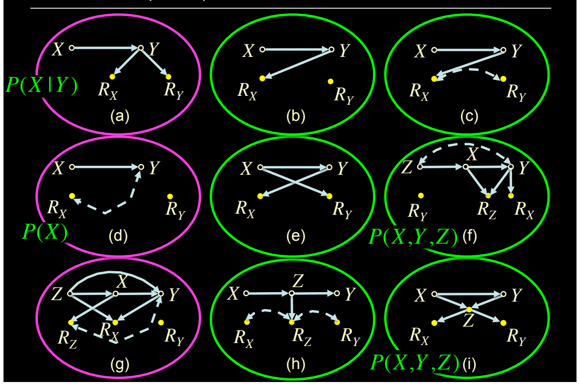
Non-recoverability
Theoretical impediment to any estimation strategy

Testability
Given a model G, when does it have testable implications (refutable by some partially-observed data D')?

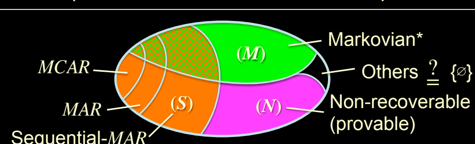
What is known about Recoverability and Testability?

MCAR	recoverable	almost testable
MAR	recoverable	uncharted
MNAR	uncharted	uncharted

IS $P(X,Y)$ RECOVERABLE?



THE RECOVERABILITY PIE
(and what's in it for the user)

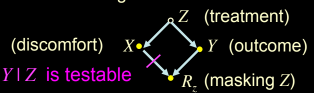


- Recoverability in *MAR* and *MCAR* models can be achieved by model-blind estimators (e.g., *MI* or *EM*).
- In areas (*M*) and (*S*), recoverability requires model-smart estimators.
- Testability – charted over the entire terrain.

THE PECULIAR CHARACTER OF TESTABILITY IN MISSING DATA

- Not all that looks testable is testable:
- Some testable implications of fully recovered distributions are not testable from missing data.

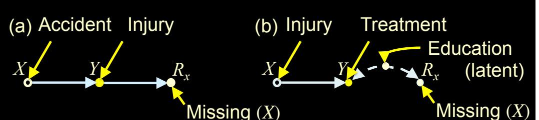
Example:



Now $X \perp\!\!\!\perp Y \mid Z$ is testable

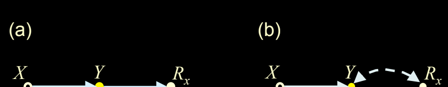
- $P(X, Y, Z)$ is recoverable, and advertises the conditional independence $X \perp\!\!\!\perp Y \mid Z$, which is falsifiable, hence testable.
- Yet $X \perp\!\!\!\perp Y \mid Z$ is not falsifiable by any data in which Z is partially missing.
- Any such data, even when generated by a model in which $X \perp\!\!\!\perp Y \mid Z$ is false, may be construed as if generated by the model above, in which $X \perp\!\!\!\perp Y \mid Z$ is true.

AN IMPOSSIBILITY THEOREM FOR MISSING DATA



- Two statistically indistinguishable models, yet $P(X,Y)$ is recoverable in (a) and not in (b).
- No universal algorithm exists that decides recoverability (or guarantees unbiased results) without looking at the model.

A STRONGER IMPOSSIBILITY THEOREM



- Two statistically indistinguishable models, $P(X)$ is recoverable in both, but through two different methods:

  In (a): $P(X) = \sum_y P(Y)P(X \mid Y, R_x = 0)$, while

  in (b): $P(X) = P(X \mid R_x = 0)$
- No universal algorithm exists that produces an unbiased estimate whenever such exists.

THE PROBLEM OF SELECTION BIAS

- Systematic exclusion of samples from the data is a major obstacle to valid causal and statistical inferences;
- In general, it cannot be removed by randomized experiments and can hardly be detected in either experimental or passive observations.

Goal: Provide methods capable of mitigating and sometimes eliminating this bias.

(Joint work with Bareinboim & Tian)

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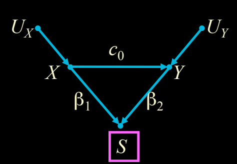
SAMPLE SELECTION IN THE LANGUAGE OF GRAPHS

Augmented graph



$S = 1$: Included in the sample
$S = 0$: Excluded from the sample

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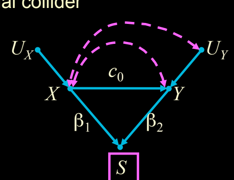
TWO SOURCES OF SELECTION BIAS:

$X \rightarrow S \leftarrow Y$ Collider
$X \rightarrow Y \leftarrow U_Y$ Virtual collider



- Cannot be eliminated by adjustment or by randomization

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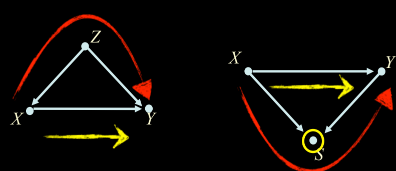
CONFOUNDING BIAS vs SELECTION BIAS

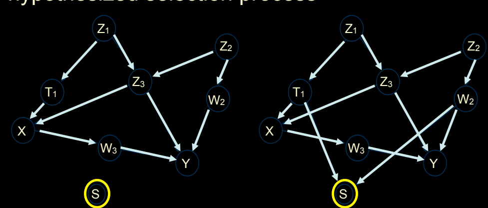
- Unblockable "flow" of information between treatment and outcome — spurious correlation.



G1 G2

THE GENERAL SELECTION BIAS PROBLEM

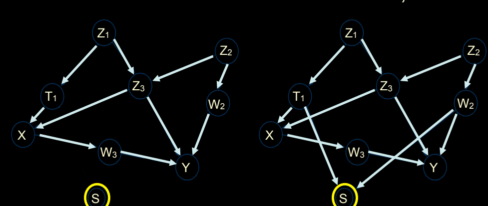
Input: An augmented causal graph, describing a hypothesized selection process



•Under what conditions can we estimate the query (e.g., $P(y \mid x)$) from $P(v \mid S = 1)$).

THE SELECTION BIAS PROBLEM

- Selection bias, caused by preferential exclusion of samples from the data, is a major obstacle to valid causal and statistical inferences;

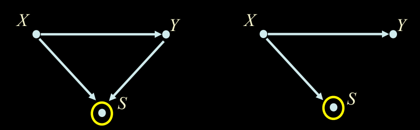


•Under what conditions can we estimate the query (e.g., P(y | x)) from P(v | S = 1).

RECOVERING FROM SELECTION BIAS

Question: Under what conditions can we estimate the distribution $P(y \mid x)$ from $P(v \mid S=1)$.

Theorem: $Q = P(y \mid x)$ is recoverable from selection biased data if and only if $(S \perp\!\!\!\perp Y \mid X)_G$.



$P(y \mid x)$ is not recoverable. $P(y \mid x)$ is recoverable.

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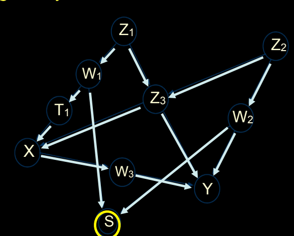
SELECTION WITH EXTERNAL INFORMATION

Estimate $Q = P(y \mid x)$ from selection biased data



Q is not recoverable by the previous theorem…
but what if $P(W_1, W_2)$ is available?

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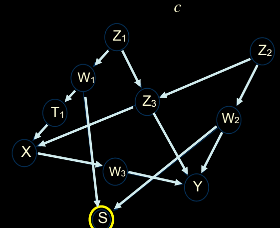
RECOVERABILITY WITH EXTERNAL INFORMATION

Theorem. $P(y \mid x)$ is recoverable if there is a set C such that $(Y \perp\!\!\!\perp S \mid C, X)$ holds in G and $P(C, X)$ is estimable.
Moreover, $P(y \mid x) = \sum_c P(y \mid x, c, S = 1)\, P(c \mid x)$



$C = \{W_1, W_2\}$? yes
$= \{W_2, Z_1, Z_2\}$? no
$= \{W_2, Z_3\}$? yes

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SUMMARY OF SELECTION BIAS RESULTS

- Nonparametric recoverability from selection bias can be decided provided that an augmented causal graph is available.
- When recoverability is feasible, the estimand can be derived in polynomial time.
- The result is complete for pure recoverability and sufficient for recoverability with external information.
- The back-door criterion can be generalized to handle selection bias.
- Stronger results can be obtained for the OR recoverability.

CONCLUSIONS

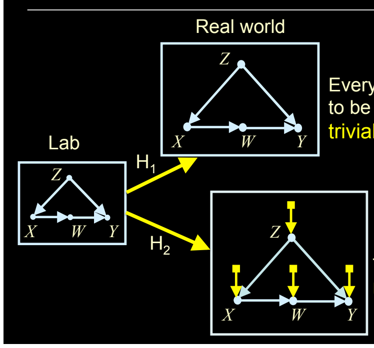
1. Think nature, not data, not even experiment.
2. Think hard, but only once – the rest is mechanizable.
3. Speak a language in which the veracity of each assumption can be judged by users, and which tells you whether any of those assumptions can be refuted by data.
4. Proceed in a language in which your research question can be answered from the assumptions plus the data.

Thank you

TRANSPORTABILITY OF KNOWLEDGE ACROSS DOMAINS (with E. Bareinboim)

1. A Theory of causal transportability
   When can causal relations learned from experiments be transferred to a different environment in which no experiment can be conducted?
2. A Theory of statistical transportability
   When can statistical information learned in one domain be transferred to a different domain in which
   a. only a subset of variables can be observed? Or,
   b. only a few samples are available?

MOVING FROM THE LAB TO THE REAL WORLD . . .



Everything is assumed to be the same, trivially transportable!

Everything is assumed to be the different, not transportable!

MOTIVATION

WHAT CAN EXPERIMENTS IN LA TELL ABOUT NYC?

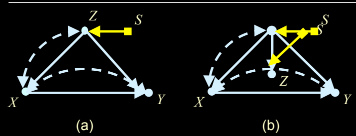
Experimental study in LA
Measured: $P(x,y,z)$
$P(y \mid do(x),z)$

Observational study in NYC
Measured: $P^*(x,y,z)$
$P^*(z) \neq P(z)$

Needed: $P^*(y \mid do(x)) = \; ? \; = \sum_z P(y \mid do(x),z) P^*(z)$

Transport Formula (calibration): $F(P, P_{do}, P^*)$

TRANSPORT FORMULAS DEPEND ON THE STORY



S ■⟶ Factors producing differences

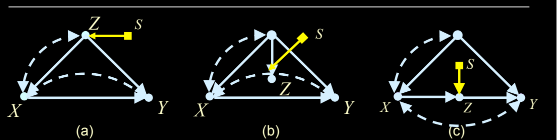
a) ***Z*** represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

b) ***Z*** represents language skill

$$P^*(y \mid do(x)) = \ ?P(y \mid do(x))$$

TRANSPORT FORMULAS DEPEND ON THE STORY



a) ***Z*** represents age

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) P^*(z)$$

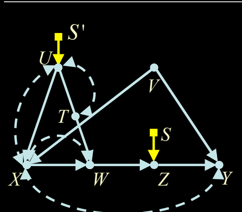
b) ***Z*** represents language skill

$$P^*(y \mid do(x)) = P(y \mid do(x))$$

c) ***Z*** represents a bio-marker

$$P^*(y \mid do(x)) = \ ?\sum_z P(y \mid do(x), z) P^*(z \mid x)$$

GOAL: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE



INPUT: Annotated Causal Graph
S ⟶ Factors creating differences

OUTPUT:
1. Transportable or not?
2. Measurements to be taken in the experimental study
3. Measurements to be taken in the target population
4. A transport formula

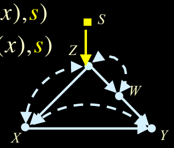
$$P^*(y \mid do(x)) = f[P(y, v, z, w, t, u \mid do(x)); P^*(y, v, z, w, t, u)]$$

TRANSPORTABILITY REDUCED TO CALCULUS

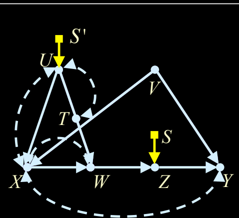
Theorem

A causal relation R is transportable from $\prod$ to $\prod^*$ if and only if it is reducible, using the rules of *do*-calculus, to an expression in which S is separated from *do*( ).

$$
\begin{aligned}
R(\textstyle\prod^*) &= P^*(y \mid do(x)) = P(y \mid do(x), s) \\
&= \sum_w P(y \mid do(x), s, w) P(w \mid do(x), s) \\
&= \sum_w P(y \mid do(x), w) P(w \mid s) \\
&= \sum_w P(y \mid do(x), w) P^*(w)
\end{aligned}
$$



RESULT: ALGORITHM TO DETERMINE IF AN EFFECT IS TRANSPORTABLE



INPUT: Annotated Causal Graph
S → Factors creating differences

OUTPUT:
1. Transportable or not?
2. Measurements to be taken in the experimental study
3. Measurements to be taken in the target population
4. A transport formula
5. Completeness (Bareinboim, 2012)

$$P^*(y \mid do(x)) = \sum_z P(y \mid do(x), z) \sum_w P^*(z \mid w) \sum_t P(w \mid do(w), t) P^*(t)$$

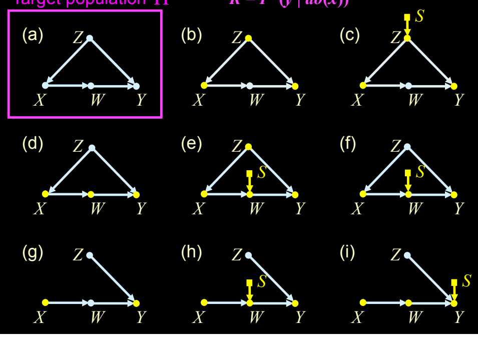
FROM META-ANALYSIS TO META-SYNTHESIS

The problem

How to combine results of several experimental and observational studies, each conducted on a different population and under a different set of conditions, so as to construct an aggregate measure of effect size that is "better" than any one study in isolation.

META-SYNTHESIS AT WORK

Target population $\prod^*$ $R = P^*(y \mid do(x))$



META-SYNTHESIS REDUCED TO CALCULUS

Theorem
$\{\prod_1, \prod_2, \ldots, \prod_K\}$ – a set of studies.
$\{D_1, D_2, \ldots, D_K\}$ – selection diagrams (relative to $\prod^*$).
A relation $R(\prod^*)$ is "meta estimable" if it can be decomposed into terms of the form:

$$Q_k = P(V_k \mid do(W_k), Z_k)$$

such that each Q_k is transportable from D_k.

MATHEMATICAL RESULT #3: (Transportability and meta-transportability are solved)

- Nonparametric transportability of experimental results from multiple environments can be decided in polynomial time, provided commonalities and differences are encoded in selection diagrams.
- When transportability is feasible, the transport formula can be derived in polynomial time, which specifies the information needed to be extracted from each environment to synthesize a consistent estimate for the target environment.
- The algorithm is complete.

DETERMINING CAUSES OF EFFECTS
A COUNTERFACTUAL VICTORY

- Your Honor! My client (Mr. A) died BECAUSE he used that drug.

- Court to decide if it is MORE PROBABLE THAN NOT that A would be alive BUT FOR the drug!

$PN = P(? \mid A \text{ is dead, took the drug}) \geq 0.50$

THE ATTRIBUTION PROBLEM

Definition:

1. What is the meaning of $PN(x,y)$:
   "Probability that event y would not have occurred if it were not for event x, given that x and y did in fact occur."

Answer:

$$PN(x,y) = P(Y_{x'} = y' \mid x, y)$$

Computable from M

THE ATTRIBUTION PROBLEM

Definition:

1. What is the meaning of $PN(x,y)$:
   "Probability that event y would not have occurred if it were not for event x, given that x and y did in fact occur."

Identification:

2. Under what condition can $PN(x,y)$ be learned from statistical data, i.e., observational, experimental and combined.

ATTRIBUTION MATHEMATIZED
(Tian and Pearl, 2000)

- Bounds given combined nonexperimental and experimental data ($P(y,x)$, $P(y_x)$, for all y and x)

$$\max\left\{\begin{array}{c} 0 \\ \frac{P(y)-P(y_{x'})}{P(x,y)} \end{array}\right\} \leq PN \leq \min\left\{\begin{array}{c} 1 \\ \frac{P(y'_{x'})}{P(x,y)} \end{array}\right\}$$

- Identifiability under monotonicity (Combined data)

$$PN = \frac{P(y \mid x) - P(y \mid x')}{P(y \mid x)} + \frac{P(y \mid x') - P(y_{x'})}{P(x,y)}$$

CAN FREQUENCY DATA DECIDE LEGAL RESPONSIBILITY?

	Experimental		Nonexperimental	
	$do(x)$	$do(x')$	x	x'
Deaths (y)	16	14	2	28
Survivals (y')	984	986	998	972
	1,000	1,000	1,000	1,000

- Nonexperimental data: drug usage predicts longer life
- Experimental data: drug has negligible effect on survival
- Plaintiff: Mr. A is special.
  1. He actually died
  2. He used the drug by choice
- Court to decide (given both data):
  Is it more probable than not that A would be alive but for the drug?

$$PN \triangleq P(Y_{x'} = y' \mid x, y) > 0.50$$

SOLUTION TO THE ATTRIBUTION PROBLEM

- WITH PROBABILITY ONE $1 \leq P(y'_{x'} \mid x,y) \leq 1$
- Combined data tell more that each study alone